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Sums of Two Polygonal Numbers in Rings

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Abstract of Poster Presentation: In 1640, Fermat wrote a letter to Mersenne regarding a question about when a natural number can be expressed as a sum of two squares. Thanks to Euler, we fully understand the answer to this question in $\mathbb{Z}$. April 2nd 2014, Harrington, Jones, and Lamarche published a paper that explains sufficient and necessary set of conditions for when every element in the ring $\mathbb{Z}_n$ can be expressed as a sum of two squares. Our research motivates from the realization that square numbers are just a specific type of polygonal number; namely 4-gonal numbers. We carried out our research with the goal of finding the sufficient and necessary set of conditions for when every element in $\mathbb{Z}_{n\geq 2}$ can be expressed as a sum of two s-gonal numbers while (1) allowing and (2) not allowing 0 as a summand. Most of the work was first done in $\mathbb{Z}_{p^\alpha}$, with prime p, using tools in algebraic number theory such as properties of quadratic residues; modular arithmetic; and also combinatorics. Then by applying the Chinese Remainder Theorem, we were able to generate relevant conditions for $\mathbb{Z}_n$.

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