Young Mathematicians Conference 2018
*The Ohio State University, August 10-12*

# The Scaling Function Connection Problem in the Two-Dimensional Ising Model

**William P Warner** (wipawa@umich.edu)
*University of Michigan* [Mentor:Thomas Bothner]

Abstract of Poster Presentation: The Ising model, introduced by physicists Wilhelm Lenz and Ernst Ising in 1920, is a mathematical model of ferromagnetism. States of the two-dimensional Ising model consist of random, interacting spins $\sigma_{ij} = \pm 1$ at each grid point in a finite box $\Lambda = \{(i,j) \in \mathbb{Z} \mid 0 \leq i \leq M, 0 \leq j \leq N\}$. Unlike in the one dimensional case, this model exhibits a phase transition at a critical temperature $T_c > 0$, famously quantified by Onsager and Kaufmann in 1949. A follow up breakthrough occurred in 1977 when Wu, McCoy, Tracy, and Barouch exactly calculated the spin-spin correlation functions of the model in the limit $\Lambda \uparrow \mathbb{Z}^2$ and $T \to T_c$. Their result introduced Painlevé functions into mathematical physics and the underlying connection problem for the limiting correlation function (i.e. relating its asymptotic expansions in different directions to each other) became of central importance and a standing problem in the field. The first solution to it was obtained by Craig Tracy in 1991 based on operator theoretical arguments. We solve a generalization of Tracy's problem by solely relying on techniques of Hamiltonian dynamical systems combined with known asymptotic expansions for Painlevé functions.

Received: July 18, 2018