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EIGENVALUE DISTRIBUTION OF KRONECKER PRODUCTS OF RANDOM MATRICES

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Abstract of Poster Presentation: Random matrix theory has wide applications from physics (modeling the energy level of heavy nuclei) to number theory (describing the distribution of zeros of the Riemann zeta and more generally L -functions). One of the central results is Wigner Semi-Circle Law, which states that the distribution of normalized eigenvalues of a real symmetric or complex Hermitian matrix with entries randomly chosen from converges to the semicircle density. Past work has extensively studied the distribution of eigenvalues from random matrix ensembles whose entries possess additional structure, which can lead to very different behavior for the density of states.

While Random Matrix Theory has successfully modeled many properties of zeros of L -functions, there are some operations in number theory which to date have not had analogous formulations in Random Matrix Theory. In particular, through Rankin-Selberg convolution one can take two L -functions and create a new L -function. We investigate a possible random matrix analogue to Rankin-Selberg convolution, the Kronecker product.

Specifically, we consider the effect on the distribution of eigenvalues under the Kronecker product \otimes , where $A \otimes B$ is formed by replacing each entry a_{ij} of A by the block $a_{ij}B$. This product has the property that the spectrum of the matrix $A \otimes B$ is the term-by-term product of the spectra of A and B . This allows us to take two matrix ensembles whose eigenvalue distributions are understood, and combine them via the Kronecker product into a family with a new distribution. We use this approach to build families of matrices exhibiting behavior that is hybrid between previously studied behaviors, and which can possibly model similar behavior in number theory. Time permitting, we also share results on several natural generalizations of the Kronecker product, including the Tracy-Singh product on block matrices.

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