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Linear Recurrence Relations with Non-constant Coefficients and Benford's Law

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Abstract of Report Talk: In 1938 physicist Frank Benford observed that in many numerical datasets, the leading digit is not equidistributed among $\{1, \ldots, 9\}$ as one might expect, but instead heavily biased towards low digits, particularly 1. In many situations the probability of a number having first digit d base b is $\log_b(1 + \frac{1}{d})$ (so base 10 it ranges from about 30% for a first digit of 1, down to about 4.6% for a leading digit of 9); this phenomenon became known as Benford's Law. In addition to being of theoretic interest, Benford's Law has found applications in numerous fields from data integrity (used to detect tax and data fraud) to computer science (designing optimal systems to minimize rounding errors).
One of the central questions in the subject is determining what systems should follow Benford's law. It has long been known that sequences generated by linear recurrence relations with constant coefficients obey Benford's Law. We extend this result to new families of linear recurrences with non-constant coefficients and non-linear recurrences. We prove that, for certain families of functions f and g, a sequence generated by a recurrence relation of the form $a_{n+1} = f(n)a_n + g(n)a_{n-1}$ is Benford for all initial values. The proof proceeds by parameterizing the coefficients to get a recurrence relation of lower degree, and then converting to a new parameter space. From there we show that for suitable choices of f and g where $\frac{g(n)}{f(n)^2} \to 0$ as $n \to \infty$, the main term dominates and the behavior is equivalent to equidistribution problems previously studied. As time permits, we will describe our efforts at generalizing these results further to multiplicative recurrence relations and higher-order recurrence relations with non-constant coefficients.

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