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Deformations of the Weyl Character Formula via Ice Models

Alexander M Vetter (avetter@villanova.edu)
Alexandra I Embry (aiembry@iu.edu)
University of Minnesota [Mentor:Benjamin Brubaker]

Abstract of Report Talk: We study problems in combinatorial representation theory related to the Weyl character formula for complex Lie groups. In particular, we study highest weight characters of $SO(2n+1)$ using statistical mechanics. We explore properties of tetravalent graphs known as ice models, or six-vertex models, using the Yang-Baxter equation and similar relations. In particular, we show these ice models are in bijection with a variant of Gelfand-Tsetlin patterns, which are in turn in bijection with certain shifted Young Tableaux. We then define a partition function over the admissible states of an ice model from a highest weight $\lambda = (\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 1)$. These partition functions are shown to correspond to a certain deformation of the Weyl character formula for Cartan type B. Inspired by techniques for enumerating alternating sign matrices by Kuperberg, we show our partition function is a symmetric function. This is achieved by choosing Boltzmann weights that satisfy the Yang-Baxter equation; these weights correspond to the so-called "free fermion point" of the model. Our work is inspired by Tokuyama, Hamel-King, and Okada who gave deformations of highest weight characters via patterns and tableaux in certain classical types and Brubaker and Schultz who gave deformations via ice models.

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