

EDGE-COLORED GRAPHS AND NILPOTENT LIE ALGEBRAS

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Abstract of Report Talk: Computing the automorphism group of a nilpotent Lie algebra is of great importance in dynamics and Riemannian geometry. A Lie algebra is *2-step nilpotent* if all n -fold Lie brackets are trivial for all $n \geq 3$, and we restrict our attention to this class. Such nilpotent Lie algebras can often be constructed from combinatorial data given by graphs. The automorphism groups of 2-step nilpotent Lie algebras associated with finite simple graphs were studied by Dani-Mainkar (2005) to give examples of nilmanifolds admitting Anosov automorphisms which give rise to interesting dynamical systems.

In this project, we consider a class of 2-step nilpotent Lie algebras associated with edge-colored directed graphs as described by Ray (2016) and Payne-Schroeder (2017), and study the structure of their automorphism groups. Some of these automorphisms, which we call *graph Lie automorphisms*, arise from special graph automorphisms of the directed edge-colored graph, which form a subgroup of the group of *color permuting automorphisms* of the corresponding undirected graph. We study an interesting family of edge-colored directed graphs and compute their graph Lie automorphism groups and color permuting automorphism groups. These graphs are complete graphs with a prime number of vertices and uniformly colored edges. We show that if the number of vertices is p , then the graph Lie automorphism group is isomorphic to the dihedral group of order $2p$, and the order of the color permuting automorphism group of the corresponding undirected graph is $p(p-1)$.

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