Gaussian Behavior in Zeckendorf Decompositions Arising From Lattices

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Abstract of Report Talk: Zeckendorf’s Theorem states that any positive integer can be written uniquely as a sum of non-adjacent Fibonacci numbers; interestingly, one can also define the Fibonacci numbers as the unique sequence for which every positive integer has a unique decomposition as the sum of non-adjacent terms. Much is known about the Zeckendorf decomposition; for example, as \( n \to \infty \) the distribution of the number of summands of integers in \([F_n, F_{n+1}]\) converges to a Gaussian, and for each \( m \in [F_n, F_{n+1}] \) almost surely the distribution of gaps between summands converges to a geometric random variable with parameter \( 1/\phi \) (where \( \phi \) is the Golden Mean).

We consider a two-dimensional lattice analogue, where a legal decomposition of a number \( n \) is a collection of lattice points such that each point is included at most once, once a point is chosen all future points must have smaller \( x \) and smaller \( y \) coordinates, and the sum of the values of the points chosen equals \( n \). We prove that the distribution of the number of summands in these lattice decompositions converges to a Gaussian distribution. If time permits we will discuss the distribution of the gaps and further generalizations to arbitrary \( d \) (once \( d > 2 \) we no longer have a closed form expression for the number of legal paths of length \( k \) starting at the point \((n, n, \ldots, n)\) on the lattice, and must resort to a delicate asymptotic analysis and generating functions).

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