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Gaussian Behavior in Zeckendorf Decompositions Arising From Lattices

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Abstract of Report Talk: Zeckendorf's Theorem states that any positive integer can be written uniquely as a sum of non-adjacent Fibonacci numbers; interestingly, one can also define the Fibonacci numbers as the unique sequence for which every positive integer has a unique decomposition as the sum of non-adjacent terms. Much is known about the Zeckendorf decomposition; for example, as $n \to \infty$ the distribution of the number of summands of integers in $[F_n, F_{n+1})$ converges to a Gaussian, and for each $m \in [F_n, F_{n+1})$ almost surely the distribution of gaps between summands converges to a geometric random variable with parameter $1/\phi$ (where ϕ is the Golden Mean).
We consider a two-dimensional lattice analogue, where a legal decomposition of a number n is a collection of lattice points such that each point is included at most once, once a point is chosen all future points must have smaller x and smaller y coordinates, and the sum of the values of the points chosen equals n. We prove that the distribution of the number of summands in these lattice decompositions converges to a Gaussian distribution. If time permits we will discuss the distribution of the gaps and further generalizations to arbitrary d (once $d > 2$ we no longer have a closed form expression for the number of legal paths of length k starting at the point $(n, n, \ldots, n)$ on the lattice, and must resort to a delicate asymptotic analysis and generating functions).

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