Young Mathematicians Conference 2018
The Ohio State University, August 10-12

Unweighted Contraction-Deletion of Stanley's Symmetric Chromatic Polynomial

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Abstract of Report Talk: In a landmark paper in 1995, Stanley defined a symmetric function version of the chromatic polynomial of a graph. For a tree graph, it happens that the regular chromatic polynomial is completely determined by the number of vertices. Whereas, Stanley conjectured that his polynomial completely distinguishes non-isomorphic trees. Currently, there is a contraction-deletion identity for the symmetric chromatic polynomial which depends on using graphs with weighted vertices. This, however, doesn't allow one to use the principles of induction through unweighted graphs where we induct over the number of vertices.

We introduce a new way to represent weights that allows us to construct a novel contraction-deletion identity on the symmetric chromatic polynomial of unweighted graphs. This identity allows us to take the symmetric chromatic polynomial of any graph and reduce it to the algebraic combination of the polynomials of star-shaped graphs. We give a more general form of the contraction-deletion method which can represent the difference between two graphs' symmetric chromatic polynomials in terms of the differences in those graphs.

These methods of contraction-deletion allow us to further explore the combinatorial relationship among different representations of Stanley's polynomial such as Newton's identities on symmetric bases.

[Joint work with Rushil Raghavan, Eric Fawcett, Jake Huryn] Received: July 23, 2018