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Minimizing Properties of $(1/k)$-Geodesics on Doubled Polygons

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Abstract of Report Talk: A $(1/k)$-geodesic on a metric space M is a closed geodesic $\gamma : \mathcal{S}^1 \to M$ that is length-minimizing on all subintervals of length $l(\gamma)/k$. The minimizing index, $minind(M)$, is the smallest $k \in \mathbb{N}$ such that M admits a $(1/k)$-geodesic. The central focus of our work is the minimizing indices of doubled regular n-gons, X_n.
For even n, $minind(X_n) = 2$, and for odd n, we conjecture that $minind(X_n) = 2d$, where d is the smallest prime factor of n. We prove that $minind(X_3) = 6$ and classify many families of geodesics on X_n. All of these geodesics pass very close to vertices, an indicator of high minimizing index. Our main result is a necessary and sufficient condition for a sequence $n_i \in \mathbb{N}$ to satisfy $minind(X_{n_i}) \to \infty$: that for any subsequence n_{i_j} and any $k \in \mathbb{N}$, there is no sequence of $(1/k)$-geodesics on $X_{n_{i_j}}$ that converges to a closed geodesic on a doubled disk. Such n_i would imply a sequence of smooth manifolds has unbounded minimizing index while the Gromov-Hausdorff limit has minimizing index 2.

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