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Symmetric Chromatic Polynomial for Signed Graphs

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Abstract of Report Talk: In a landmark 1995 paper, Stanley introduces a symmetric generalization of the chromatic polynomial of a graph, X_G. By encoding information regarding the multiplicity of a color in any given coloring, the symmetric chromatic polynomial proved to be a much stronger graph invariant than the original chromatic polynomial. This sparked several notable conjectures and other work regarding its enumerative and algebraic properties. In this vein, we first introduce a new, simpler proof of a previously known weighted contraction-deletion relation on X_G, which no longer relies on an expression in the power sum basis of symmetric functions. From there, we consider a B_n-type symmetric generalization of the chromatic polynomial of a signed graph. We then develop a new weighted-contraction deletion identity for the signed symmetric chromatic polynomial. Notably, this identity relies on the introduction of ordered-pair weights on the vertices of a graph, similar to the integer weights observed in the unsigned case. Moreover, we introduce two other methods of computing the symmetric chromatic polynomial of signed graphs, one considering subsets of the edge set and the other considering partitions of the vertex set. Finally, the techniques that yield these generalizations can be naturally adjusted to a symmetric chromatic polynomial of a voltage graph, yielding similar formulae and results.

[Joint work with James Enouen, Ishaan Shah]

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