

Young Mathematicians Conference 2018
The Ohio State University, August 10-12

DISTRIBUTION OF PRIMES IN NUMBER FIELDS

Maria R Ross (mrross@pugetsound.edu)
Casimir M Kothari (ckothari@princeton.edu)
SMALL REU Williams College [Mentor:Steven Miller]

Abstract of Report Talk: A Gaussian prime is a prime element in the ring of Gaussian integers $\mathbb{Z}[i]$, which takes the form $a + bi$ with $a^2 + b^2$ a rational prime congruent to 1 modulo 4, or p , where p is a rational prime congruent to 3 modulo 4. Viewing the Gaussian primes as points on the complex plane, one may ask about the statistical properties of the angles of these points. In 1919, Hecke showed that these angles are uniformly distributed as p varies. Motivated by a model from random matrix theory, Rudnick and Waxman conjectured a formula for their variance. For a quadratic number field K , similar questions can be asked about the prime elements in the ring of integers \mathcal{O}_K . However, since the ring of integers \mathcal{O}_K forms a lattice in the \mathbb{Q} -vector space K , there are two distinct ways to interpret the elements of \mathcal{O}_K in this scenario: we may view them as points in the complex plane or as points in the lattice. This gives rise to two possible definitions of the angle of a prime element. We apply the L -functions ratios conjecture, a recipe to predict the behavior of averages of ratios of L -functions, to Hecke L -functions to determine the variance of the distribution of primes. We generalize previous work from $\mathbb{Q}[i]$ to all imaginary quadratic fields with class number 1. Finally, we discuss the correct way to view the elements of the ring of integers to produce a well-defined notion of angle.

[Joint work with Trajan Hammonds, Ben Logsdon]

Received: July 19, 2018