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ANALOGS OF POLYNOMIALS ON THE SIERPINSKI GASKET FOR FAMILIES
OF LAPLACIANS

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Abstract of Report Talk: The Sierpinski Triangle or Sierpinski Gasket (SG) can be thought of as the simplest nontrivial example of a fractal which supports a theory of differential calculus, based on the Laplacian constructed by Kigami, which is the analog of the second derivative on the unit interval. With this differential calculus comes a theory of polynomials on SG (solutions to $\Delta^n u = 0$ for some n) studied by Needleman, Strichartz, Teplyaev, and Yung. We generalize this theory of polynomials to a recently constructed one-parameter family of self-similar symmetric Laplacians developed by Fang, King, Lee, and Strichartz. We define a basis for the space of polynomials corresponding to each of these new Laplacians, the monomials, characterized by the property that a certain “derivative” is 1 at one of the boundary points, while all other “derivatives” vanish. These monomials are analogous to $\frac{x^k}{k!}$ on the interval. We compute the values of the monomials at the boundary points of SG (the analog of the coefficients $\frac{1}{k!}$) as functions of the parameter for the family of Laplacians. We obtain relationships between ratios of coefficients and certain Neumann eigenvalues of the Laplacians. Surprisingly, the results for the general case are quite different from the results for the Kigami Laplacian.

[Joint work with Christian Loring]

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