

GRAPH-RESTRICTED DECOMPOSITIONS: A FURTHER GENERALIZATION OF
ZECKENDORF'S THEOREM

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Abstract of Report Talk: Zeckendorf's theorem states that every natural number can be decomposed as a sum of distinct non-adjacent Fibonacci numbers; moreover, the Fibonacci numbers are the unique sequence in which every natural number has a unique decomposition, following these rules. We generalize this interplay between sequences and legal decompositions by choosing a graph G with vertices \mathbb{N} (seen as indices), and forbidding two terms to be used together in a decomposition if their indices are connected by an edge (we continue to insist that the terms are distinct). We prove there is a canonical choice of sequence in which every natural number has a G -restricted decomposition; however, such decompositions are no longer always unique. We give sufficient criteria for uniqueness of decomposition, and classify sequences for which a simple procedure can construct a graph which generates them.

Our framework is sufficiently general that nearly every past generalization of Zeckendorf's theorem and legal decompositions can be easily described as a special case. Previous work has focused on the distribution of number of summands used in various decompositions. Using the graph perspective, we extend such results and prove that a large class of graphs produce distributions which are Gaussian. If time permits we will discuss when to expect this typical behavior, as well as what else can arise.

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