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Local h polynomials and the monodromy conjecture

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Abstract of Report Talk: For a polynomial $f \in \mathbb{Z}[x_1, x_2, \ldots, x_n]$, let $N_m(f)$ denote the number of solutions to $f \equiv 0 \pmod{p^m}$ for a fixed prime p. The generating series $\sum_{i=1}^{\infty} N_m(f)t^m$, known to be rational function, is called the *Igusa-Poincaré series*. Igusa's monodromy conjecture predicts that the singularities of f control the poles of the Igusa-Poincaré series. More precisely, if p is a pole of the Igusa-Poincaré series, then $\exp(2\pi i \mathrm{Re}(p))$ is an eigenvalue of the monodromy transformation of the Milnor fibration at some point in the singular locus of f. Assuming f is Newton nondegenerate, both the poles and eigenvalues have combinatorial formulas in terms of the Newton polyhedron of f, i.e. the upper convex hull in $\mathbb{R}^n$ of the exponents of the monomials with non-zero coefficients. To each facet of the Newton polyhedron, one naturally associates both a candidate pole p and a contribution to the multiplicity of $\exp(2\pi i \mathrm{Re}(p))$ as an eigenvalue of monodromy. The contribution to this multiplicity is the value of a combinatorially defined polynomial with non-negative integer coefficients at $(1, \ldots, 1)$. In particular, it is strictly positive unless this combinatorially defined polynomial, which is a relative version of Stanley's local h polynomial, vanishes identically. By studying combinatorial conditions that are necessary for the vanishing of these relative local h-polynomials, we prove several new cases of Igusa's monodromy conjecture.

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