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Expected Chromatic Number of Random Subgraphs

Catherine Lee (catherine.lee@yale.edu)
Henry Reichard (henry.reichard@yale.edu)
Yale University [Mentor:Patrick Devlin]

Abstract of Report Talk: Given a graph G and $p \in [0,1]$, we obtain a probability distribution of subgraphs, G_p, by keeping all the vertices of G and including each edge independently with probability p. When $G = K_n$ this distribution is called the Erdős-Rényi random graph and is denoted $G(n,p)$. The chromatic number $\chi(G)$ is the minimal number of colors needed to color all the vertices of G such that no adjacent vertices are the same color. Bollobás (1988) showed that $\mathbb{E}[\chi(G(n,1/2))] \sim \frac{n}{2\log_2 n}$; however, for arbitrary G the best current bounds, due to Alon et al. (1997), are $C_p \frac{\chi(G)}{\log|V(G)|} \leq \mathbb{E}[\chi(G_p)] \leq \chi(G)$, where $C_p \leq 1$. We discuss possible refinements to the lower bound on $\mathbb{E}[\chi(G_p)]$.
Bukh asks whether the lower bound can be improved to $\mathbb{E}[\chi(G_p)] \geq C_p \frac{\chi(G)}{\log \chi(G)}$. Shinkar (2018) answers Bukh's question affirmatively for a large family of graphs and asks whether his method can be extended to answer the question for all graphs. We show that it cannot, providing the Kneser graphs as a family of counterexamples. Shinkar also proposes an alternative lower bound, $\mathbb{E}[\chi(G_p)] \geq \chi(G)^p$. We demonstrate that this bound is incorrect for general $p \in [0,1]$ as it fails for odd cycles, though it is valid for $p = 1/n$ where $n \in \mathbb{N}$. Finally, we consider alternative approaches to answering Bukh's question.

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