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THE SYMMETRY INDEX OF A GRAPH AND FAMILIES OF ASYMMETRIC  
GRAPHS

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**Abstract of Report Talk:** A graph is *asymmetric* if it has a trivial automorphism group. Asymmetric graphs were first studied by Erdős and Rényi in 1963. We define the *symmetry index* of a graph, denoted  $si(G)$ , to be the minimum of  $r + s$  so that the resulting graph is asymmetric, where  $r$  is the number of edges removed from a graph and  $s$  is the number of edges added to a graph. We show that when  $n \geq 8$ ,  $si(P_n) = 1$ ,  $si(C_n) = 2$ ,  $si(C_{n^2 \pm 1}(1, n)) = 2$ ,  $si(K_{1, n-1}) = 2n - 9$ ,  $si(P_n + tP_1) = t$ ,  $si(C_n + tP_1) = t + 1$ ,  $si(C_{n^2 \pm 1}(1, n) + tP_1) = t + 1$ , and  $\lfloor \frac{6n}{7} \rfloor \leq si(K_n) \leq n - 2$ .

Erdős and Rényi also posed the question of finding the number of asymmetric trees. We give a partial result, showing that the number of asymmetric subdivided stars is approximately  $q(n - 1) - \lfloor \frac{n-1}{2} \rfloor$  where  $q(n)$  yields the number of ways to sum to  $n$  using distinct positive integers, found by Hardy and Ramanujan in 1918.

In addition, we investigate  $k$ -regular asymmetric Hamiltonian graphs and determine infinite families for  $k = 3$  and  $k = 4$ . Furthermore, we show the existence of  $k$ -regular asymmetric Hamiltonian graphs for each  $k > 6$ .

[Joint work with Alejandra Brewer Castano and Adam Gregory]

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