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SATO-TATE IN ARITHMETIC PROGRESSIONS AND LEHMER'S CONJECTURE

**Trajan A Hammonds** (thammond@andrew.cmu.edu)

**Casimir M Kothari** (ckothari@princeton.edu)

*SMALL REU Williams College* [Mentor: Jesse Thorner]

**Abstract of Poster Presentation:** In 1916, Ramanujan studied the values of his tau function, which govern fluctuations about the main term in approximating the number of ways to represent a prime as a sum of 24 squares. In 1947, Lehmer conjectured that the Ramanujan tau function,  $\tau(n)$ , is nonzero for all  $n$ . The tau function also appears as the coefficients of the modular discriminant, which is the prototypical example of a cuspidal newform. A modular form  $f(z)$  of level  $N$  on a congruence subgroup  $\Gamma$  is a cusp form if it vanishes at the cusps of  $\Gamma$ ; this forces the constant coefficient of its Fourier expansion to equal 0. Intuitively, we say that the cusp form  $f(z)$  is a newform if it does not appear at a lower level than  $N$ . It follows from Deligne's proof of the Weil Conjectures that the coefficients of a newform with weight  $k$  are bounded in absolute value by  $2p^{(k-1)/2}$ , hence we can write them as  $a_f(p) = 2p^{(k-1)/2} \cos(\theta_p)$ , for some angle  $\theta_p \in [0, \pi]$ . The Sato-Tate conjecture, proven by Barnet-Lamb, Geraghty, Harris, and Taylor, states that these angles are equidistributed with respect to the measure  $d\mu_{ST} = \frac{2}{\pi} \sin^2(\theta) d\theta$ . Let  $f(z) \in S_k^{\text{new}}(\Gamma_0(N))$  be a newform with squarefree level  $N$  without complex multiplication. In 2017, Rouse and Thorner proved an explicit version of the Sato-Tate conjecture under assumptions on the symmetric power  $L$ -functions of  $f$  which include the Generalized Riemann Hypothesis. They use this explicit result to bound the density of nonzero coefficients of the Ramanujan tau function below by  $1 - 1.54 \times 10^{-13}$ . Under the same assumptions, we prove an explicit version of the Sato-Tate conjecture for primes in arithmetic progressions, and use this result to improve the lower bound found by Rouse and Thorner.

[Joint work with Noah Luntzlara, Hunter Wieman, Steven J Miller]

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