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Sato-Tate in Arithmetic Progressions and Lehmer's Conjecture

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Abstract of Poster Presentation: In 1916, Ramanujan studied the values of his tau function, which govern fluctuations about the main term in approximating the number of ways to represent a prime as a sum of 24 squares. In 1947, Lehmer conjectured that the Ramanujan tau function, $\tau(n)$, is nonzero for all n. The tau function also appears as the coefficients of the modular discriminant, which is the prototypical example of a cuspidal newform. A modular form $f(z)$ of level N on a congruence subgroup Γ is a cusp form if it vanishes at the cusps of Γ; this forces the constant coefficient of its Fourier expansion to equal 0. Intuitively, we say that the cusp form $f(z)$ is a newform if it does not appear at a lower level than N. It follows from Deligne's proof of the Weil Conjectures that the coefficients of a newform with weight k are bounded in absolute value by $2p^{(k-1)/2}$, hence we can write them as $a_f(p) = 2p^{(k-1)/2}\cos(\theta_p)$, for some angle $\theta_p \in [0, \pi]$. The Sato-Tate conjecture, proven by Barnet-Lamb, Geraghty, Harris, and Taylor, states that these angles are equidistributed with respect to the measure $d\mu_{ST} = \frac{2}{\pi}\sin^2(\theta)\,d\theta$. Let $f(z) \in S_k^{\text{new}}(\Gamma_0(N))$ be a newform with squarefree level N without complex multiplication. In 2017, Rouse and Thorner proved an explicit version of the Sato-Tate conjecture under assumptions on the symmetric power L-functions of f which include the Generalized Riemann Hypothesis. They use this explicit result to bound the density of nonzero coefficients of the Ramanujan tau function below by $1 - 1.54 \times 10^{-13}$. Under the same assumptions, we prove an explicit version of the Sato-Tate conjecture for primes in arithmetic progressions, and use this result to improve the lower bound found by Rouse and Thorner.

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