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Anti-Power Subwords of the Thue-Morse Word

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Abstract of Report Talk: An essential concept in combinatorics on words is that of a k-power, which is a word of the form w^k for some word w. The study of k-powers gave rise to what is now famously known as the Thue-Morse word $\mathbf{t}$, an infinite binary word originally introduced by Axel Thue in 1912. Recently, Fici, Restivo, Silva, and Zamboni introduced the notion of a k-anti-power, which is defined as a word of the form $w_1 w_2 \cdots w_k$, where $w_1, w_2, \ldots, w_k$ are distinct words of the same length. For an infinite word x and a positive integer k, define $AP_j(x, k)$ to be the set of all integers m such that $x_{j+1} x_{j+2} \cdots x_{j+km}$ is a k-anti-power, where x_i denotes the i-th letter of x. Define also $\mathcal{F}_j(k) = (2\mathbb{Z}^+ - 1) \cap AP_j(\mathbf{t}, k)$. For all $k \in \mathbb{Z}^+$, $\gamma_j(k) = \min(AP_j(\mathbf{t}, k))$ is a well-defined positive integer, and for $k \in \mathbb{Z}^+$ sufficiently large, $\Gamma_j(k) = \sup((2\mathbb{Z}^+ - 1) \setminus \mathcal{F}_j(k))$ is a well-defined odd positive integer. In his 2018 paper, Defant shows that $\gamma_0(k)$ and $\Gamma_0(k)$ grow linearly in k. We generalize Defant's methods to prove that $\gamma_j(k)$ and $\Gamma_j(k)$ grow linearly in k for any nonnegative integer j. In particular, we show that $1/10 \leq \liminf_{k\to\infty}(\gamma_j(k)/k) \leq 9/10$ and $1/5 \leq \limsup_{k\to\infty}(\gamma_j(k)/k) \leq 3/2$. Additionally, we show that $\liminf_{k\to\infty}(\Gamma_j(k)/k) = 3/2$ and $\limsup_{k\to\infty}(\Gamma_j(k)/k) = 3$.

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