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All Ramsey-critical graphs and star-critical Ramsey numbers for cycles versus K_5

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Abstract of Report Talk: For graphs F, G, and H, if all red-blue edge colorings of F contain either red G or blue H as a subgraph, then we write $F \to (G, H)$. The Ramsey number for graphs G and H, denoted $R(G, H)$, is the smallest integer s such that $K_s \to (G, H)$. It is known that $R(C_n, K_5) = 4n - 3$ for $n \geq 5$. We prove that for all $n \geq 5$, any graph on $4n - 4$ vertices which does not contain C_n or an independent set of order 5 contains $4K_{n-1}$, and thus we characterize all Ramsey-critical graphs for C_n versus K_5. The graph $K_{s-1} \sqcup K_{1,t}$ is constructed by adding a vertex to K_{s-1} and joining it to t of its vertices. The star-critical Ramsey number $r_*(G, H)$ is defined as the minimum t such that $K_{s-1} \sqcup K_{1,t} \to (G, H)$, where $s = R(G, H)$. Values of $r_*(C_n, K_m)$ are known for $m \in \{3, 4\}$. In this work, we extend this to $m = 5$ and some cases for $m = 6$, and we present computational proofs of small cases and a computer-free proof of the general result for $n \geq 8$ and $m = 5$. We also compile a survey of known star-critical Ramsey numbers involving simple graphs such as cycles, paths, and fans.

[Joint work with Jacob Liddy]

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