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PLAYING BALL WITH THE LARGEST PRIME FACTOR

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Abstract of Report Talk: In 1974 Hank Aaron hit his 715th major league homerun, beating Babe Ruth's previous record of 714. Due to this event the numbers 714 and 715 received quite a bit of publicity, and it was observed that they have some interesting properties. The first one noticed is that their product is the product of the first 7 primes (conjectured to be the largest consecutive pair of numbers to be the product of the first k primes for some k). However, these numbers have another interesting property. If we let $S(n)$ be the sum of the prime factors of n taken with multiplicity, then $S(714) = S(715)$, and in their honor we now call n with $S(n) = S(n + 1)$ a Ruth-Aaron number. The first few such n are 5, 8, 15, 77, 125, 714, 948 and 1330. Carl Pomerance proved that if one assumes Schinzel's Hypothesis H then there are infinitely many solutions to $S(n) = S(n + 1)$, and later he and Paul Erdős proved that the Ruth-Aaron numbers have density 0 and the sum of their reciprocals converges (as the sum of the reciprocals of the primes converge, this means there are significantly fewer such numbers up to any x as $x \rightarrow \infty$). The key ingredients in their argument are results about the largest prime factors of integers, which is of interest and importance in numerous other problems in number theory; thus this and related problems provide motivation and testing grounds for advanced results on prime divisors.

Building on their methods, we extend their results by replacing S with other functions f on the prime factors (weighted with multiplicity); the sequence of such n and the arithmetic inputs in the proof depend greatly on the choice of f . For example, if $f(n)$ is the sum of the k -th powers of the prime factors of n then there are no solutions to $f(n) = f(n + 1)$ for integer $k \in \{2, \dots, 10\}$ if $n \leq 1000000$. The first few n for the squaring function ($k = 2$) are 6371184, 16103844, and 49214555, while if f is the Euler totient function the first few n occur significantly earlier at 3, 80, 175, 272, 492, and 860. If time permits, we will discuss generalizations to other arithmetic functions and how to handle the resulting number theoretic obstructions.

[Joint work with n/a]

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