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INTERSECTING SUBGRAPH FAMILIES

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Abstract of Report Talk: The Erdos-Ko-Rado Theorem (1961) is an important result in extremal combinatorics regarding the maximal size of intersecting families of sets. A natural extension is to intersecting subgraph families. We denote by $2^{\binom{n}{2}} f(H, n)$ the maximal size of a family of subgraphs of K_n such that any pairwise intersection contains a copy of H . Ellis, Filmus, and Friedgut (2010) showed that $f(K_3, n) = \frac{1}{8}$ and that fixing a triangle junta is uniquely optimal. Christofides (2012) proved that $f(P_3, n) \geq \frac{17}{128}$, but whether this bound is tight remains unproven. Little else has been explored in the area of intersecting graph families.

We approach questions concerning other intersecting subgraph families with techniques including probabilistic graph theory, entropy, and discrete Fourier analysis. We prove that if $\lim_{n \rightarrow \infty} f(H, n) = \frac{1}{2}$, then $\lim_{n \rightarrow \infty} f(\bigcup_i^\ell H, n) = \frac{1}{2}$ even for ℓ increasing, as long as $\ell = o(\sqrt{n})$. We extend a weaker version of this result to fractional family sizes less than one-half, improving the bound for $\bigcup_i^\ell H$ -intersecting families of size $\frac{1}{3} \leq f(H, n) < \frac{1}{2}$. We also bound the sizes of maximal families of graphs whose intersections have particular chromatic number, k components, or k -connectivity via induction and spanning tree subgraphs. We furthermore introduce a measure of “subgraph complexity” and seek to determine whether, as complexity increases for a subgraph of ℓ edges, $f(H, n) \rightarrow 2^{-\ell}$ (the fractional size of the H -junta construction). Finally, we conjecture that $f(\text{connected}, n) = \frac{1}{2^{n-1}}$, and discuss partial progress towards this result.

[Joint work with Patrick Devlin, Michael Doppelert, Harish Vemuri]

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