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Polyhedral Circle Packings

Alisa F Cui (alisa.cui@yale.edu)
Rutgers [Mentor:Alex Kontorovich]

Abstract of Poster Presentation: A circle packing in $\mathbb{R}^2$ is a collection of circles whose interiors are disjoint and whose union fills space densely. Circle packings have been studied since at least the early 18th century by Leibniz, who wrote of the classical Apollonian gasket. The Apollonian packing is constructed from three mutually tangent circles, with further circles added such that they are tangent to three others in the packing. However, this is not the only way to construct a circle packing. The Koebe-Andreev-Thurston Theorem asserts that every polyhedron and its dual can each be realized as a collection of circles (cluster and cocluster respectively); the Kontorovich-Nakamura Structure Theorem allows us to generate a circle packing by inverting the cluster about the cocluster. By examining the bend ($\frac{1}{radius}$) of each circle in the packing, we can determine whether a packing is integral; that is, if the circles have exclusively integer bends. This question is of fundamental interest, particularly because packings associated with polyhedra are not well documented. This work classifies circle packings associated with polyhedra on up to 7 vertices, identifying a set of fundamental integral polyhedra that cannot be decomposed by slicing along faces or edges.

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