Young Mathematicians Conference 2018
The Ohio State University, August 10-12

The Numerical Range of a Composition Operator on the Hardy Space on the Unit Ball

Laney T Bowden (lbowden1@rams.colostate.edu)
Ellie N Lochner (ellie.lochner@snc.edu)
University of Michigan-Dearborn [Mentor:John Clifford]

Abstract of Report Talk: For a bounded operator T on a Hilbert Space $\mathbb{H}$, the numerical range of T is the subset $W(T)$ of $\mathbb{C}$ given by $W(T) = \{< Tx, x > : ||x|| = 1\}$. We study the numerical range of the composition operator, C_A, on the Hardy space $H^2(\mathbb{B}_n)$ where A is an $n \times n$ matrix that is a self-map of the unit ball. We show the set of homogeneous holomorphic polynomials of degree k is a reducing subspace for C_A; it follows that $W(A) \subseteq W(C_A)$. In the special case where A is a weighted shift, $W(C_A) = \text{convex hull}(W(A) \cup \{1\})$. We completely characterize the numerical range of the operator when A is unitarily similar to a Jordan-normal form that maps the ball to the ball by decomposing our operator into the direct sum of shifts and normal operators.

[Joint work with Julia Balukonis, Fatme Hourani]

Received: July 18, 2018