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Spectral Theory of Perturbed Kohn Laplacians on Spheres

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Abstract of Report Talk: A CR-manifold is a submanifold in $\mathbb{C}^M$ with extra structure stipulating that the dimension of the complex part of its tangent space is pointwise invariant under some complex structure map. The Kohn Laplacian $\Box_b$ is a second order differential operator intrinsically defined on any CR-manifold. The spectrum of $\Box_b$ reveals important information about the embeddability of abstract CR-manifolds.

In the particular case of the Rossi sphere $(\mathbb{S}^3, \mathcal{L}^t)$, where $\mathbb{S}^{2N-1}$ denotes the standard $(2N-1)$ real dimensional unit sphere in $\mathbb{C}^N$, showing that 0 belongs to the essential spectrum of $\Box_b$ is enough to conclude that $(\mathbb{S}^3, \mathcal{L}^t)$ is not embeddable into $\mathbb{C}^M$ for any $M > 0$. Furthermore, when the CR structure is induced from the ambient manifold $\mathbb{C}^M$ (unperturbed operator), the spectrum of $\Box_b$ is explicitly computed on any sphere $\mathbb{S}^{2N-1} \subset \mathbb{C}^M$ by Folland. On the contrary, the Kohn Laplacian induced by abstract CR structures on high dimensional CR-manifolds is less easily understood. One effective approach in this setting is to study the Kohn Laplacian on the subspaces of spherical harmonics in $L^2(\mathbb{S}^{2N-1})$. When restricted to the finite dimensional subspaces of spherical harmonics, $\Box_b$ can be expressed as a matrix, and one can either explicitly compute or at least obtain bounds on the eigenvalues.

In this project, we expand on previous work and study asymptotics of the spectrum on the Rossi sphere, and we provide sharp upper and lower bounds on the maximum eigenvalues on the invariant subspaces by invoking the Gershgorin Circle Theorem. As for the spheres in higher dimensions (when $N > 3$), we utilize *Mathematica* to compute eigenvalues of perturbed Kohn Laplacians and implement Folland's results in this setting. Additionally we tie our computations to a theorem of Boutet de Monvel, which proves that when $N > 3$ any strongly pseudoconvex CR structure on $\mathbb{S}^{2N-1}$ is in fact embeddable into $\mathbb{C}^M$ for some M. In ongoing work we focus on $\Box_b$ on $\mathbb{S}^5$ induced by perturbed CR structures similar to the Rossi perturbation on $\mathbb{S}^3$, and we use *SymPy* to investigate its spectrum. We also work to verify Boutet de Monvel Theorem in this setting by demonstrating the essential spectrum of the perturbed Kohn Laplacians is strictly positive.

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