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Abstracts of Presentations

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Plenary Lectures

Putting Topology to Work

Robert Ghrist
University of Pennsylvania

Abstract of Lecture:
Mathematics implicates motions and machines; computations and colorings; the strings and arrows of life. Perhaps the grandest expression of the beauty and power of Mathematics is revealed in the quantification and qualification that which is not there: holes.

Topology — the mathematics of holes — will be surveyed with a fresh look at the many ways in which topology is used in data management, networks, and sensing applications.

Train tracks, braids, and surface homeomorphisms

Dan Margalit
Georgia Institute of Technology

Abstract of Lecture: Suppose you want to stir a pot of soup with several spoons. What is the most efficient way to do this? Thurston’s theory of surface homeomorphisms gives us a concrete way to analyze this question.

First, we will explain how the different mixing patterns can be encoded via mathematical braids. Then, to each mixing pattern we can associate a real number called the entropy. This number gives a first approximation to the amount of mixing that is happening.

We will start from scratch with a simple example, state the Nielsen-Thurston classification of surface homeomorphisms, and give some open questions about entropies of general surface homeomorphisms.

Long-time behavior of dispersive equations

Monica Visan
University of California, Los Angeles

Abstract of Lecture: We will examine how a trichotomy of behaviors can occur for a dispersive equation, namely, soliton formation, wave collapse (think Big Crunch), and scattering.
**Universal Law for the Distribution of Odd Periodic Cycles within Chaos in Nonlinear Dynamical System**

Almas Abdulla  
*Florida Institute of Technology*  
[Mentor: Ugur Abdulla]

**Abstract of Poster Presentation:** This paper reveals a new universal transition law from chaos to odd periodicity in nonlinear logistic type discrete equations:

\[ x_{n+1} = 4\lambda x_n(1 - x_n) \]

We reveal the following universal law for the distribution of the odd periodic windows:

\[
\text{Chaos} \rightarrow 17 \rightarrow 13 \rightarrow 15 \rightarrow 11 \rightarrow 13 \rightarrow 9 \rightarrow 11 \rightarrow 7 \rightarrow 9 \rightarrow 5 \rightarrow 7 \rightarrow 3 \rightarrow \text{Chaos} \quad (0.1)
\]

Every superstable odd cycle of period larger than 7 appears twice in this diagram (0.1). To understand the non-monotonic nature of the distribution of odd cycles, we attempt to present a fine classification of odd cycles by employing symbolic dynamics and representing each odd cycle with a cyclic permutation and an oriented graph of transitions. All the superstable odd cycles have a universal cyclic permutation and oriented graph of transitions. Moreover, for any odd integer \( k \geq 7 \), the cyclic permutation and the oriented graph of one of the two superstable k-cycles (second one from right) in diagram (0.1) has a first-kind cyclic permutation and transition graph according to the Sharkovsky theory. They are called minimal odd orbits with simple cyclic permutation and oriented graph of transitions which is implied by the fact that the map has odd cycle of period \( k \), but no odd cycles of smaller period. It is revealed that the other superstable \( k \)-cycles have different cyclic permutations and transition graphs, which we call second-kind cyclic permutations and transition graphs. This paper demonstrates that the second-kind transition graph of the \( k \)-cycle is a superposition of the transition graphs of the two \( k - 2 \)-cycles with first kind transition graphs. It is conjectured that the second-kind transition graph of the \( k \)-cycle is a unique transition graph, (with the exception of the inverse transition graph) which is implied by the fact that the map has odd cycles of periods \( k \) and \( k - 2 \), but no odd cycles of smaller period. Finally, numerical analysis of the Duffing equation with periodic external force demonstrates that the transition from chaotic strange attractor to odd periodic limit cycles is relevant. However, it seems the Duffing oscillator does not inherit the intriguing non-monotonic distribution of the odd periodic cycles within chaos relevant for discrete models.

[AA29130031]

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**Multi-soliton solution to the modified Korteweg-de Vries equation**

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[Mentor: Tuncay Aktosun]

**Abstract of Poster Presentation:** The modified Korteweg-de Vries (mKdV) equation $u_t + 6u^2u_x + u_{xxx} = 0$ is a nonlinear partial differential equation used to model traffic flow and congestion along highways as well as internal waves in stratified fluids. Besides its importance in applications, the mKdV equation is mathematically significant because it has certain solutions, known as multi-soliton solutions, that can be expressed in terms of elementary functions. An $n$-soliton solution contains $n$ individual particle-like entities that interact with each other only when they are close. In a recent method, the $n$-soliton solution to the mKdV equation is expressed in terms of three constant matrices $A, B, C$ of sizes $n \times n$, $n \times 1$, $1 \times n$, respectively, as $u(x, t) = -2CE^{-1}B$, where $E := e^{2Ax - 8A^3t} + Pe^{-2Ax + 8A^3t}P$ and $P$ is the $n \times n$ matrix satisfying the auxiliary matrix equation $AP + PA = BC$. Our primary goal in this research is to directly verify that the solution formula for the mKdV equation holds for any positive integer $n$. We investigate the unique solvability of the auxiliary matrix equation, verify the solution formula for the mKdV equation, and relate the eigenvalues of the matrix $A$ to various physical properties of the individual solitons in the $n$-soliton solution.

[AJ15191230]

[Joint with Gabriel Gonzales]

**Heuristic Optimal Control on Polynomial Dynamical Systems Expedited by Use of Algebraic Geometry**

**Hussein Al-Asadi**

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[Mentor: Franziska Hinkelmann]

**Abstract of Poster Presentation:** Polynomial Dynamical Systems (PDS), finite dynamical systems in which the transition of each variable is described by a polynomial, form a mathematical basis for many discrete models used in systems biology. Our novel mathematical contribution is combining heuristic search methods and algebraic geometry to conduct efficient optimal control on PDS. Specifically, we provide an implementation of an adaptive genetic algorithm to find optimal control for gene regulatory networks. If the control objective depends on the long term behavior of the system, we solve a system of polynomial equations instead of calculation by enumeration, which is computationally infeasible on large networks. Solving a system of polynomial equations is a long studied problem in algebraic geometry for which we use Gröbner bases. We demonstrate the feasibility of our algorithm by applying it to a network of proteins involved in the cell cycle transition, our algorithm confirms the optimum found in the published results and was computed in a matter of seconds. Our implementation is available through a web-based software tool (http://adam.vbi.vt.edu/).

[AH24224933]

[Joint with Atsuya Kumano, Laurel Ohm, Alice Toms, Reinhard Laubenbacher (co-mentor), Matt Oremland (co-mentor)]

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On the Properties of Variations of a Fibonacci Type Polynomial Sequence

Brandon Alberts (albert43@msu.edu)
Michigan State University [Mentor: Aklilu Zeleke]

Abstract of Report Talk: Several properties of a Fibonacci type polynomial sequence, given by the recurrence
\[ F_1(x) = -a, \quad F_2(x) = x - a, \quad \text{and} \quad F_n(x) = x^k F_{n-1}(x) + F_{n-2}(x) \]
have been studied in [1],[2],[3]. Here \( a \) and \( k \) are positive integers. In particular, the asymptotic behavior of the maximum roots for \( k = 1 \), \( a \) arbitrary and \( a = 1 \), \( k \) arbitrary have been established.

In this talk we introduce a quasi-Fibonacci polynomial sequence defined by the recursion
\[ F_1^q(x) = -1, \quad F_2^q(x) = x - 1, \quad \text{and} \quad F_n^q(x) = F_{n-1}^q(x) + x^k F_{n-2}^q(x). \]
Here \( k \) is a real number. We present several interesting properties such as an explicit formula for \( F_n^q \) and prove that, for every \( n \), \( F_n^q \) has no rational maximum roots. We will also establish asymptotic results for the maximum roots of \( F_n^q \). Moreover, we will present asymptotic results for the maximum roots of the regular Fibonacci polynomial sequence \( F_n \) and its derivatives when \( k = 2 \) and \( a \) is an arbitrary real number. Comparisons between the regular Fibonacci polynomial sequence and the Quasi Fibonacci polynomial sequence will be made.

References

[AB2211918] [Joint with Rebbecca Miller and Laquinta Stuart]

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The Distribution of Generalized Ramanujan Primes

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Williams College [Mentor: Steven Miller]

Abstract of Report Talk: In 1845, Bertrand conjectured that for all integers \( x \) greater than or equal to 2, there exists at least one prime in \((x/2, x]\). This was proved by Chebyshev in 1860, and then generalized by Ramanujan in 1919, who showed for any integer \( n \) there is a prime \( R_n \) such that \( \pi(x) - \pi(x/2) \geq n \) for all \( x \geq R_n \). We generalize the interval of interest by introducing a parameter \( c \in (0, 1) \) and defining the \( n \)-th \( c \)-Ramanujan prime \( R_{c,n} \) as the smallest integer such that for all greater integers \( x \), there are at least \( n \) primes between \( cx \) and \( x \).

Using consequences of strengthened versions of the Prime Number Theorem, we prove the existence of \( R_{c,n} \) for all \( n \) and all \( c \), that the asymptotic behavior is \( R_{c,n} \sim p_{1+m/n}^m \) (where \( p_m \) is the \( m \)-th prime), and that the percentage of primes that are \( c \)-Ramanujan converges to \( 1-c \). We then study finer questions related to their distribution among the primes, and see that the \( c \)-Ramanujan primes display striking behavior, deviating significantly from a probabilistic model based on biased coin flipping. This model is related to the Cramer model, which correctly predicts many properties of primes on large scales but has been shown to fail in some instances on smaller scales.

[AN23162912] [Joint with Olivia Beckwith]

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Groups with a Base Property Analogous to That of Vector Spaces

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[Mentor: R. Keith Dennis]

Abstract of Report Talk: A generating set for a finite group, G, is said to be irredundant iff no proper subset of the generating set generates G. Diaconis and Saloff-Coste (1996) demonstrated that the efficacy of certain algorithms for the random generation of group elements depend upon the minimum and maximum size of irredundant generating sets. Intriguingly, these values also constrain the group structure.

In a forthcoming paper, McDougall-Bagnall and Quick define a finite group, G, to be a B group iff all irredundant generating sets of G have the same length. B stands for “base”, since this property is analogous to the property that every basis of a given vector space has the same cardinality. The authors ask whether B groups are solvable and whether quotients of B groups are also B groups. I have resolved both questions in the affirmative and have classified B groups.

Topology of Graphic Hyperplane Arrangements

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Northern Arizona University  
[Mentor: Michael Falk]

Abstract of Report Talk: A complex hyperplane arrangement is a finite set, \( A = \{H_1, \cdots, H_n\} \), of codimension one linear subspaces, hyperplanes, \( H_i \), of \( \mathbb{C}^\ell \). We are interested in studying the topology of the complement in complex projective space, \( M = \mathbb{C}P^{\ell-1} - \bigcup_{H \in A} H \), of graphic arrangements, arrangements associated with a simple graph. The main algebraic object associated with an arrangement, the Orlik-Solomon algebra, \( \mathcal{A}(A) \), is purely determined by the underlying combinatorics of the graph and is isomorphic to the cohomology ring of the complement, \( H^*(M) \). With the goal of characterizing \( M \) up to homotopy type, we study invariants of the Orlik-Solomon algebra, particularly the first resonance variety. Specifically, we prove a formula for determining the dimension of the span of the resonance variety. We describe the polymatroid determined by the components of the first resonance variety, at least in the case where the underlying graph has no 4-cliques. Using this we produce a pair of parallel-indecomposable, irreducible, inerectible graphic arrangements whose Orlik-Solomon algebras have the same quadratic closure.

[Joint with Donald S. Mathers, Dr. Michael J. Falk, Caleb Holtzinger]

Pathological Orbits Of Subspace-Hypercyclic Operators

Alexander A Azzam  
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[Mentor: Blair Madore]

Abstract of Summary Talk: An operator \( T \) on a Hilbert space \( \mathcal{H} \) is said to be \( \mathcal{M} \)-hypercyclic for a proper closed subspace \( \mathcal{M} \) if there exists a vector whose orbit under \( T \) is dense in \( \mathcal{M} \). The study of linear chaos has been motivated by its potential to solve the invariant subspace problem. Quite surprisingly, despite their rigid structure, the dynamics of linear operators can be fantastically complex. For example, a beautiful theorem of Bourdon and Feldmen show that if \( \text{Orb}(T, x) \) is somewhere dense in \( \mathcal{H} \), then \( \text{Orb}(T, x) \) is everywhere dense. What is less predictable, however, is the behavior of an operator in the proper subspaces of \( \mathcal{H} \). In this talk, we'll show that the Bourdon-Feldmen theorem is false in the context of subspace hypercyclicity, and demonstrate how pathological the orbits of subspace-hypercyclic operators can be.
**Simplicial matrix-tree theorem and a polynomial invariant for triangulations**

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The Ohio State University [Mentor: Sergei Chmutov]

**Abstract of Report Talk:** A classical matrix-tree theorem expresses the cofactors of the Laplacian matrix for a graph as a sum over all spanning trees. This was generalized to simplicial complexes by A. Duval, C. Klivans, and J. Martin using the Laplacian matrix for the corresponding chain complex. Another classical invariant for graphs is the Tutte polynomial. V. Krushkal and D. Renardy generalized the Tutte polynomial to simplicial complexes using triangulations. In the case for graphs, the cofactors of the Laplacian were shown to give the free term of the Tutte Polynomial. In seeking an analogous relation, we modify the Krushkal-Renardy polynomial so that the free term is a constant multiple of the cofactor of the Laplacian for complexes. We prove that this modification satisfies the same duality as the original Krushkal-Rendary polynomial. Additionally we discuss analogues of contraction and deletion for simplicial complexes.

**Gaps between summands in generalized Zeckendorf decompositions**

Olivia D Beckwith (obeckwith@gmail.com)
Williams College [Mentor: Steven Miller]

**Abstract of Report Talk:** Zeckendorf proved any integer can be written uniquely as a sum of non-adjacent Fibonacci numbers. This was recently generalized by Miller-Wang, who replaced the Fibonacci numbers with the terms of any recurrence relation of the form $A_{n+1} = c_1 A_n + c_2 A_{n-1} + \cdots + c_L A_{n+1-L}$ (under suitable restrictions on the $c_i$’s). They proved that the number of summands in these generalized Zeckendorf decompositions for integers in $[A_n, A_{n+1})$ converges to a Gaussian as $n \to \infty$.

We examine the distribution of gaps between the indices of the summands in these decompositions, specifically for the binary expansions ($B_n = 2B_{n-1}$), and the Fibonacci numbers ($F_n = F_{n-1} + F_{n-2}$). We prove many results about the gaps. For example, in the binary case we derive formulas for the largest expected gap and the expected number and variance of smallest gaps, and for the Fibonacci case prove that the probability of a gap of length $2+m$ ($m \geq 0$) is $\frac{\phi^{-1}}{\phi} \phi^{-m}$, with $\phi$ the golden mean. Our methods are combinatorial, involving generating functions and recurrences.

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Distributions of Eigenvalues of Weighted, Structured Matrix Ensembles

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Williams College [Mentor: Steven Miller]

Abstract of Report Talk: The distribution of eigenvalues of random matrices has many applications (nuclear physics, number theory, network theory). The universality in behavior is striking, and is often related to Central Limit Theorem type results. For example, in quantum mechanics the fundamental equation is \( H\Psi_n = E_n\Psi_n \) (\( H \) is the Hamiltonian, \( \Psi_n \) the energy eigenstate with eigenvalue \( E_n \)). Though \( H \) is too complicated to diagonalize, a typical \( H \) behaves similarly to the average behavior of the ensemble of matrices where each independent entry is chosen independently from some fixed probability distribution. Depending on the physical system, the matrix \( H \) is constrained. The most common constraints are \( H \) is real-symmetric (where the limiting spectral measure is the semi-circle) or Hermitian. Studies have determined the limiting spectral measures for many structured ensembles, such as Toeplitz and circulant matrices. These systems have very different behavior; the limiting spectral measures for both have unbounded support. Given a structured ensemble, we introduce a parameter to continuously interpolate between these two behaviors. We fix a \( p \in [1/2, 1] \) and study the ensemble of signed structured matrices by multiplying the \( ij^{th} \) and \( ji^{th} \) entries of a matrix by a randomly chosen \( \epsilon_{ij} \in \{1, -1\} \) where \( \epsilon_{ij} = 1 \) with probability \( p \). For \( p = \frac{1}{2} \), we prove that the limiting spectral measure is the semi-circle. For all other \( p \), for many structured ensembles (including the Toeplitz and circulant) we prove the measure has unbounded support, and converges to the original ensemble as \( p \to 1 \).

The proofs are by Markov’s Method of Moments. The analysis of the \( 2k^{th} \) moment for such distributions involves the pairings of \( 2k \) vertices on a circle. The contribution of each pairing in the signed case is weighted by a factor depending on \( p \) and the number of vertices involved in at least one crossing. These numbers are of interest in their own right, appearing in problems in combinatorics and knot theory. The number of configurations with no vertices involved in a crossing is well-studied and are the Catalan numbers. We discover and prove similar formulas for configurations with 4 and 6 vertices in at least one crossing. For higher-order moments, we prove closed-form expressions for the expected value and variance for the number of vertices in at least one crossing. As the variance converges towards 4, these results yield a lot of information about the limiting measure.

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Behavior of Solutions to Hamilton’s Equations under the Stäckel Transform

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Abstract of Report Talk: In classical Hamiltonian dynamics in $n$ spatial dimensions, a system is called superintegrable if it admits $2n - 1$ functionally independent constants of the motion, including the Hamiltonian energy function. These functions on the phase space, the cotangent bundle of the configuration space manifold, are polynomial in the momentum forms and invariant in time. The constants of the motion generate an algebra under the Poisson bracket operation; superintegrable systems are soluble using this algebra. The intersection of the phase space with the $2n - 1$ level sets obtained by specifying the constants of the motion is a one-dimensional trajectory in the phase space. A process called the Stäckel transform maps a superintegrable system into another superintegrable system in such a way that the structure of the Poisson algebra is preserved but the configuration space metric is altered. We show that the Stäckel process not only induces a diffeomorphism between phase spaces, but also defines an isomorphism between trajectories in the two systems. We will illustrate these concepts using pairs of physically relevant systems which are extensions of the Kepler and harmonic oscillator problems. Lastly, we will discuss the extension of our results to quantized systems.

A complete characterization of optimal vertex rankings of paths, cycles, and joins of graphs

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Abstract of Poster Presentation: A $k$-ranking of a graph is a labeling of the vertices with $\{1, 2, \ldots, k\}$ where any path between two vertices of the same label contains a vertex with a strictly larger label. These rankings have been applied to the scheduling of manufacturing systems, monitoring of communication networks, Cholesky factorization of matrices in parallel, and VLSI layout generation.

Following along the lines of the chromatic number, the rank number of a graph, $\chi_r(G)$, is defined to be the smallest $k$ such that $G$ has a $k$-ranking. In addition to $\chi_r(G)$-rankings, we investigate sum-optimal rankings, where the sum over all labels is minimized. While $k$-rankings and sum-optimal rankings have been studied, little has been done to quantify the number of these rankings. We use tools from both graph theory and combinatorics to characterize and enumerate all possible $\chi_r(G)$-rankings and sum-optimal rankings for paths, cycles, and joins of graphs. It is likely that the ideas presented in this project can be applied to larger families of graphs.
A GROUP THEORETIC PERSPECTIVE ON PEBBLE MOTION PROBLEMS

Christian Bueno  
Florida International University  
[Mentor: Miroslav Yotov]

Abstract of Summary Talk: Permutation pebble motion problems (PPM) are puzzles in which pebbles are placed on the vertices of a graph and the question of whether one arrangement can be turned into another through legal moves is considered. In this project we generalize R.M. Wilson’s definition of groups of PPM problems to all graphs with arbitrary number of empty spaces and study properties of these groups. Furthermore, where Wilson’s paper classified only the groups for 2-connected PPM puzzles with one empty space, we classify all possible groups for PPM puzzles with one empty space and classify the groups for all 2-connected graphs with arbitrary number of spaces.

Representations of String Links and Tangles

Christian Bueno  
Ohio Wesleyan University  
[Mentor: Craig Jackson]

Abstract of Poster Presentation: The string link monoid is a generalization of the braid group created by allowing the strands of braids to loop and knot. We consider two representations on string links that extend the Burau representation of the braid group. The first, due to X. S. Lin, is defined probabilistically via sums of weighted paths along strands of the link. The second is a combinatorial/topological representation defined by recursively applying the Conway skein relation to the string link, resolving it into braids on which the representation takes the familiar form of Burau. We show that these two representations agree over many nontrivial string links. These calculations support the conjecture by T. Kerler that these representations are identical. In further investigations, we consider the case of 2-strand string links in depth, relating it to the theory of rational tangles. We define a natural extension of Lin’s representation to generic tangle diagrams and compute several examples. Lastly, we give one consequence of the conjecture, namely, a formula relating the Alexander polynomial of a link closure to its matrix entries under the representation.

Negatively Curved Slab Surfaces

Clark W Butler  
Indiana University  
[Mentor: Chris Connell]

Abstract of Report Talk: We study negatively curved slab surfaces, which are complete surfaces of negative curvature that are embedded between two parallel planes in Euclidean space. There are very few known examples of such surfaces, and all known examples are homeomorphic either to a plane or an annulus. We construct new examples of negatively curved slab surfaces of these known topological types, and investigate the properties of generic negatively curved slab surfaces with the goal of a complete classification of the possible topologies of these surfaces. Our primary tools are moving planar cross sections of the surface, Morse functions, and the behavior of the asymptotic line field of the surface at infinity.
The Bernstein center of a \( p \)-adic unipotent group

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University of Michigan - Ann Arbor  
[Mentor: Mitya Boyarchenko]

Abstract of Report Talk: There is a well-developed duality theory for locally compact abelian groups, but when we consider possibly nonabelian groups the appropriate dual space no longer carries a group structure and its so-called Fell topology is rather difficult to work with. We provide three new characterizations of the Fell topology for certain totally disconnected groups (which include \( p \)-adic unipotent groups) and use these to prove that the Bernstein center of the category of smooth representations is naturally identified with the algebra of locally constant functions on the dual space.

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Near-injectivity of polynomial maps on number fields

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[Mentor: Michael Zieve]

Abstract of Report Talk: Any polynomial \( f(X) \) with rational coefficients induces a map \( \mathbb{Q} \to \mathbb{Q} \) via \( c \mapsto f(c) \). We show that this map is at most 4-to-1 over all but finitely many values. We also prove an analogous result over an arbitrary algebraic number field \( K \), where the constant 4 is replaced by a constant which depends only on the number of roots of unity in \( K \). The proof uses the Mordell conjecture (now Faltings’ theorem), the Riemann mapping theorem, solutions of parametrized families of differential equations, computations of ranks of (infinitely many) elliptic curves, the classification of finite simple groups, and various other ingredients. We also formulate a conjecture about arbitrary maps between varieties which implies both our results and the Mazur–Kamienny–Merel uniform boundedness theorem for rational torsion of elliptic curves.

[CA21122654]  
[Joint with Ruthi Hortsch]  
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Counting representations of a free group

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[Mentor: Sean Lawton]

Abstract of Report Talk: The collection of representations of a free group into a Lie group \( G \) corresponds to pointed bundles over a surface with boundary. As these are affine varieties, and thus field independent, we can ask how many points these spaces have over fields. In this paper, we will answer this question for all free groups, and when \( G \) is either \( SL_n(\mathbb{Z}_p) \) or \( GL_n(\mathbb{Z}_p) \), where \( p \) is prime. Also, we will show a counting function for the irreducible representations of the free group in \( SL_2(\mathbb{Z}_2) \), and what this tells us about arbitrary \( p \).  

[CS52233434]  
[Joint with Sean Lawton]  
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Relations Between Minimal Surface and Special Lagrangian Manifold

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Mentor: Spiro Karigiannis

Abstract of Report Talk: The classical Weierstrass representation locally encodes the data of a minimal surface in $\mathbb{R}^n$ by a holomorphic isotropic curve in $\mathbb{C}^n$. This representation allows one to see that there always exists an $S^1$ family of associated minimal surfaces. Unfortunately, not all minimal surfaces are actually area minimizing. However, the special Lagrangian submanifolds of $\mathbb{C}^n$ are indeed volume minimizing. Moreover, these submanifolds are calibrated by $\alpha = \text{Re}(\Omega)$, where $\Omega$ is the holomorphic $(n, 0)$-form on $\mathbb{C}^n$, and each such calibration gives rise to an $S^1$ family of special Lagrangian calibrations, given by $\alpha_\theta = \text{Re}(e^{i\theta}\Omega)$. Much of this summer’s research was dedicated to studying the relationship between special Lagrangian surfaces and the area minimizing minimal surfaces in $\mathbb{R}^4$. Naturally, one might want to know how these two $S^1$ families are related. It turns out that if $\alpha$ calibrates one minimal surface, then it calibrates all the members of its associated $S^1$ family. More surprisingly, all special Lagrangian calibrations are essentially the same in $\mathbb{R}^4$ in the sense that they calibrate the same submanifolds up to a rigid motion. At the present moment, we are investigating whether an oriented minimal surface that is both area minimizing and real isotropic (in the sense that $JA^\nu = \pm A^J\nu$, where $J$ is the complex structure on the tangent or normal space, and $A^\nu$ is the second fundamental form) is special Lagrangian. However, we suspect that not all real isotropic minimal surfaces are special Lagrangian.

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Rank Number of Rook’s Graphs

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Mentor: Jobby Jacob

Abstract of Poster Presentation: A $k$-ranking of a graph $G$ is a function $f : V(G) \to \{1, 2, ..., k\}$ such that if $f(u) = f(v)$ then every $u - v$ path contains a vertex $w$ such that $f(w) > f(u)$. The rank number of $G$, denoted by $\chi_r(G)$, is the minimum $k$ such that a $k$-ranking exists for $G$. Many papers have appeared in the topic of ranking, and several of them investigated the rank number of certain classes of Cartesian products. The rook’s graph, denoted by $K_n \times K_m$, is the Cartesian product of complete graphs $K_n$ and $K_m$. This graph represents the moves of a rook on an $n \times m$ chess board. This graph contains a multitude of paths between any given vertices, and we must consider all paths between two vertices to ensure a labeling satisfies the ranking condition. We will discuss our results, including an explicit formula for $\chi_r(K_n \times K_m)$ for certain $m$, as well as new bounds for $\chi_r(K_n \times K_m)$ for all $n$ and $m$, and results involving the structure of all minimal rankings of $K_n \times K_m$.

[DK25140724]
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Abstract of Report Talk: The Minkowski sum of two polytopes is the set of all pairwise sums of their points. In this project, we studied the Minkowski length \( L(P) \) of a lattice polytope \( P \), which is defined to be the largest number of non-trivial primitive segments whose Minkowski sum lies in \( P \). The Minkowski length represents the largest possible number of factors in a factorization of polynomials with exponent vectors in \( P \), and shows up in lower bounds for the minimum distance of toric codes.

A Minkowski sum of \( L \) primitive segments \( E_1 + \cdots + E_L \) that is contained in \( P \) is called a maximal decomposition of \( P \). We have proved that in dimension \( n \) a maximal decomposition of smallest volume has at most \( 2^n - 1 \) distinct segments. This was previously known for \( n = 2 \) and 3.

Let \( P \) be a 4D lattice polygon of Minkowski length \( L \). Consider a maximal decomposition \( E_1 + \cdots + E_L \subseteq P \) of smallest 4D volume. We proved that \( \det(E_i, E_j, E_k, E_l) \leq 14 \) and this bound is sharp. This extends previously known bounds of 1 and 2 in 2D and 3D correspondingly.

It can be easily shown that if \( P \) is a lattice polytope in \( \mathbb{R}^n \) with \( L(P) = 1 \), then \( P \) has at most \( 2^n \) lattice points. We were able to prove that there exists such a polytope with exactly \( 2^n \) lattice points. This result generalizes previously known examples in 2D and 3D.

[FB25114854]
[Joint with Ian Barnett]

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An exact solution formula for the half-line Korteweg-de Vries equation

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Abstract of Report Talk: The half-line Korteweg-de Vries (KdV) equation, \( u_t + u_x - 6uu_x + u_{xxx} = 0 \), is an integrable, nonlinear partial differential equation used to model surface waves in shallow, narrow canals and acoustic waves in plasmas (ionized gases). We develop a solution formula for a large class of solutions to the half-line KdV equation, which includes the so-called multi-soliton solutions. The formula uses a triplet of constant matrices \( A, B, C \) with respective sizes \( n \times n, n \times 1, \) and \( 1 \times n \), for any positive integer \( n \). The solution formula uses matrix exponentials and involves the auxiliary \( n \times n \) matrix \( P \) satisfying the auxiliary matrix equation \( AP + PA = BC \). We analyze the unique solvability of this matrix equation and provide the existence and uniqueness based on the eigenvalues of the matrix \( A \). We then prove that our solution formula satisfies the half-line KdV equation when the auxiliary matrix equation has a unique solution. We also investigate various physical properties of our solutions and relate those physical properties to the eigenvalues of the matrix \( A \).

[GS08114840]

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THE $F^*$-algebraic Formulation of Quantum Mechanics

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[Mentor: Ilya Gekhtman]

Abstract of Report Talk: There is an approach to quantum mechanics known as the $C^*$-algebraic formulation of quantum mechanics that takes as an assumption that the observables in quantum mechanics are exactly the self-adjoint elements of a separable, unital $C^*$-algebra. By the celebrated Gelfand-Naimark Theorem, this abstract $C^*$-algebra is isometrically isomorphic to a closed subalgebra of the $C^*$-algebra of bounded operators on a separable Hilbert space. This, however, is problematic, as unbounded operators are absolutely essential in quantum mechanics. To get around this problem, I introduce the notion of what I call an $F^*$-algebra, which is a generalization of a $C^*$-algebra in the same vein that a Fréchet space is the generalization of a Banach space. I explain how it is more natural to take observables as the self-adjoint elements of a separable, unital $F^*$-algebra, and I then use the Gelfand-Naimark Theorem to show that all separable, unital $F^*$-algebras are isomorphic an $F^*$-algebra of closed, densely-defined linear operators on a separable Hilbert space, thereby showing that this formulation of quantum mechanics is equivalent to the usual formulation given in most textbooks. [GJ23135131]

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Solution to the Matrix Equation $AX + X^*B = 0$

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University of California, Santa Barbara  
[Mentor: Fernando De Teran Vergara]

Abstract of Report Talk: The matrix equation $AX + X^*B = 0$, where $A \in \mathbb{C}^{m \times n}$ and $B \in \mathbb{C}^{n \times m}$ are arbitrary and $(\cdot)^*$ denotes either the transpose or the conjugate transpose, has been of interest to researchers since at least the 1960s. Despite appearing in multiple linear algebra problems, it has remained unsolved until now. The equation, $AX + X^*B = 0$, appears similar to the homogeneous Sylvester Equation, $AX + XB = 0$, which, in contrast, has been solved since the 1950s. Solving the Sylvester Equation reduces to solving the simpler equation $J_A X + X J_B = 0$, where the matrices $J_A$ and $J_B$ are the Jordan Canonical Forms of $A$ and $B$ respectively, and then recovering solutions to the original equation from solutions of this simpler one. The equation $AX + X^*A = 0$ was solved in 2011 by analogous methods, using the Canonical Form for Congruence, rather than the Jordan Canonical Form. Unfortunately, we may not assume our coefficient matrices are in either of these canonical forms when considering the equation $AX + X^*B = 0$. Instead, we must use the idea of a matrix pencil, a tool for coupling our coefficient matrices, $A$ and $B$, and then consider the Kronecker Canonical Form of matrix pencils under strict equivalence. By this method we are able to explicitly solve the equation $AX + X^*B = 0$, and compute the dimension of the solution space in terms of the Kronecker Canonical Form of the pencil associated with the pair $(A, B^*)$. [GN22190407]

[Joint with Daniel Montealegre, Rachel Spratt]

Received: August 1, 2011
MEASURABLE LI-YORKE SENSITIVITY

Jared D Hallett (jared.hallett@gmail.com)
Williams College [Mentor: Cesar Silva]

Abstract of Report Talk: The notion of chaos is often viewed as sensitivity to initial conditions. This is naturally defined in a topological setting. Recent papers have investigated notions of sensitivity on measure spaces using tools from ergodic theory. This is done by defining a class of metrics compatible with the measure space and then requiring sensitivity with respect to each such metric. In topological dynamics there is also the related notion of Li-Yorke sensitivity. A pair \((x, y)\) is Li-Yorke if \(\lim \inf d(T^n x, T^n y) = 0\) and \(\lim \sup d(T^n x, T^n y) > 0\). Here we define a measurable analog of this notion. A system is measurably Li-Yorke sensitive if for all compatible metrics the set of Li-Yorke pairs has full measure in the product space. We show that this guarantees the existence of an uncountable scrambled set, i.e., a subset of the space such that every pair of points in the set satisfies the Li-Yorke property. Scrambled sets play an important role for Li-Yorke systems in topological dynamics. We fit Li-Yorke sensitivity into a chain of mixing notions. We prove that ergodicity of the product implies Li-Yorke sensitivity and that this in turn implies (spectral) weak mixing. This implies in particular that Li-Yorke sensitivity is equivalent to weak mixing in the finite measure-preserving case. In this case we further prove that Li-Yorke sensitivity implies that almost all pairs are separated to a dense subset of the value set of the metric. We also construct a sensitive (but not necessarily Li-Yorke) metric for all conservative ergodic systems and prove that there exists no such metric for the class of periodic transformations.

[Joint with Lucas Manuelli]

CONSTRUCTING GENERALIZED SUM-DOMINANT SETS

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Williams College [Mentor: Steven Miller]

Abstract of Report Talk: Many of the biggest problems in additive number theory (such as Goldbach’s conjecture, the Twin Prime conjecture, and Fermat’s last theorem) can be recast as understanding the behavior of sums of a set with itself. A sum-dominant set is a finite set \(A \subset \mathbb{Z}\) such that \(|A + A| > |A - A|\). We expect such sets to be rare, as addition is commutative, and subtraction is not. Though it was believed that the percentage of subsets of \(\{0, \ldots, n\}\) that are sum-dominant tends to zero, in 2006 Martin and O’Bryant proved a positive percentage are sum-dominant.

We generalize their result to deal with the many different ways of taking sums and differences of a set. We first prove that \(|\epsilon_1 A + \cdots + \epsilon_k A| > |\delta_1 A + \cdots + \delta_k A|\) a positive percent of the time for all nontrivial choices of \(\epsilon_j, \delta_j \in \{-1, 1\}\). Previous approaches proved the existence of infinitely many such sets given the existence of one; however, no method existed to construct such a set. Using techniques from probability and additive number theory, we develop a new, explicit construction for one such set, and then extend to a positive percentage of sets.

We extend these results further, finding sets that exhibit different behavior as more sums/differences are taken. For example, we prove that for any \(m\), \(|\epsilon_1 A + \cdots + \epsilon_k A| - |\delta_1 A + \cdots + \delta_k A| = m\) a positive percentage of the time. We find the limiting behavior of \(kA = A + \cdots + A\) for an arbitrary set \(A\) as \(k \to \infty\) and an upper bound of \(k\) for such behavior to settle down. Finally, we say \(A\) is \(k\)-generational sum-dominant if \(A, A + A, \ldots, kA\) are all sum-dominant. Numerical searches were unable to find even a 2-generational set (heuristics indicate the probability is at most \(10^{-9}\), and almost surely significantly less). We prove the surprising result that for any \(k\) a positive percentage of sets are \(k\)-generational, and no set can be \(k\)-generational for all \(k\).

[Joint with Oleg Lazarev]
UNCERTAINTY AND INFORMATION IN TIME-FREQUENCY ANALYSIS

Suren M Jayasuriya (smj20@pitt.edu)
Texas A & M University
[Mentor: David Larson]

Abstract of Report Talk: We discuss uncertainty principles in time-frequency analysis and their connection with central ideas of information theory which were introduced by C.E. Shannon in his 1948 paper, “A Mathematical Theory of Communication”. In 1957, I. Hirschman proved an uncertainty result that related the entropies of a function and its Fourier transform. Similar to Donoho and Stark’s generalization of the classical Fourier uncertainty principle that relies on the approximate concentration of Fourier transform pairs on their supports, we formulate a new concept of approximate entropic concentration of random variables associated with the Fourier transform pair $x,\hat{x} \in \mathbb{R}^N$. We prove that the sum of the approximate entropies is bounded below by $\log_2(N) - \delta$ where $\delta$ is related to the approximation error. This work was done during the summer of 2011 in an REU program at Texas A & M University under the guidance of Dr. David Larson and Dr. Lewis Bowen.

FRAME THEORY OVER ARBITRARY FIELDS

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Texas A & M University
[Mentor: David Larson]

Abstract of Report Talk: The study of frames for vector spaces has become of great importance over the past three decades, forming the theoretical basis behind applications in signal processing and sampling theory. In 2009, Bodmann, et al. began to investigate frames over $\mathbb{Z}_2$ in “Frame Theory for Binary Vector Spaces”. Motivated by their work, we develop frame theory for finite-dimensional vector spaces over arbitrary fields $\mathbb{F}$ that may have a degenerate bilinear form. To overcome the degeneracy of the bilinear form, we introduce the characterization of an analysis frame as a frame for a vector space such that the analysis operator $\Theta : V \rightarrow \mathbb{F}^k$ defined by $\Theta(x) = (\langle x,x_1 \rangle, \langle x,x_2 \rangle, \ldots, \langle x,x_k \rangle)^T$ is injective. We establish equivalent results on vector spaces that admit an analysis frame, called analysis spaces, including a reconstruction formula, Riesz representation theorem, and existence of a dual frame pair. Defining a subspace of $V$ as a zero inner product space $ZIP(V) := \{x \in V | \langle x,y \rangle = 0 \ \forall y \in V \}$, we prove that every finite-dimensional vector space over an arbitrary field can be written as the algebraic direct sum of an analysis space and its zero inner product space. This work was completed in the summer of 2011 at the Math REU program at Texas A & M University under the direction of Dr. David Larson.
Linear Forms Over Finite Abelian Groups

Ran Ji  
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Kansas State University  
[Mentor: Craig Spencer]

Abstract of Report Talk: A 3-term arithmetic progression can be mathematically formalized as a non-trivial solution to the equation $x_1 + x_2 - 2x_3 = 0$, where a solution is trivial if $x_1 = x_2 = x_3$. The quest is to find the largest subset of a finite abelian group $G$ containing no 3-term arithmetic progression; this is equivalent to evaluating, for $\vec{r} = (1, 1, -2)$,

$$D(G) = \max\{|A| : A \subseteq G, A \text{ contains no nontrivial solution to } \vec{r} \cdot \vec{x} = 0\}.$$  

In 1993, Roy Meshulam proved that when $2 \nmid |G|$ and $\vec{r} = (1, 1, -2)$, $D(G) \leq M \frac{|G|}{(\log |G|)^\beta}$ for some $M > 0$ and $\beta > 0$.

Let $G$ be a finite abelian group, $s \geq 3$, and $\vec{r} = (1, \ldots, 1, -s)$. Our aim is to find the maximal cardinality of a set $A \subseteq G$ containing no non-trivial solution to $x_1 + \cdots + x_s - sx_{s+1} = 0$ with $x_i \in A (1 \leq i \leq s + 1)$. Let

$$d(m) = \sup_{c(H) \geq m} \frac{D(H)}{|H|},$$

where $c(H)$ is the rank of $H$. We prove that for any $n \in \mathbb{N}$, $d(n) \leq \frac{C}{n^{s-2}}$, where

$$C = \max\left\{1, \sqrt{s^2 + s \sqrt{\frac{2s-4}{\log 2} \sqrt{\frac{2s-4}{n^2}} + \frac{2s-1}{2(s-2)^2} (2s-2) - 1}^{s-2}}\right\}.$$  

[Vassiliev’s Planarity Criterion for Graphs with Cross Structure]

Andrew J Krieger  
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The Ohio State University  
[Mentor: Sergei Chmutov]

Abstract of Report Talk: In his study of the topology of knots, V. Vassiliev put forth a conjecture about the planarity of 4-regular graphs with an additional cross structure imposed on the vertices. The cross structure partitions the four half-edges meeting at each vertex into two pairs of opposite edges. A cross-embedding of a cross-graph is an embedding that respects the cross structure. Vassiliev’s conjecture states that a cross-graph fails to be cross-planar if and only if there exist two cycles crossing at exactly one vertex. This conjecture was proven by V. Manturov using chord diagrams. I suggest a different proof of the conjecture. A cross-graph is modified by adding four extra vertices around each original vertex, and extra edges are added among these new vertices to enforce the cross structure. There is then a one-to-one correspondence between the cross-embeddings of the original cross-graph and the standard embeddings of the modified graph. This allows the cross-planarity problem to be reduced to the standard problem of graph planarity, and thus Vassiliev’s conjecture can be proven by Kuratowski’s theorem.
Perfect State Transfer on Quotient Graphs

Michael P Landry (michaellandry@berkeley.edu)
SUNY Potsdam [Mentor: Christino Tamon]

Abstract of Report Talk: We study perfect state transfer in quantum walks on graphs using equitable partitions and quotient graphs. Some results we prove include:

- There exists a graph $G$ with perfect state transfer between two vertices $u$ and $v$ such that there is no automorphism on $G$ which swaps $u$ and $v$. Moreover, there exists an infinite family of such graphs. This answers an open question posed by Chris Godsil.

- For a collection of graphs $\{G_k\}$ and respective equitable partitions $\{\pi_k\}$, there is an equitable partition $\pi$ such that $\square_k(G_k/\pi_k) \cong (\square_kG_k)/\pi$. This generalizes a construction of David Feder which was obtained from a $k$-boson quantum walk on a single graph. Our construction yields new families of weighted perfect state transfer graphs whenever each graph in $\{G_k\}$ has perfect state transfer or is periodic with a common time.

Our proofs employ rudimentary tools from algebraic graph theory.

[LM22190351]

[Joint with Rachel Bachman, Eric Fredette, Jessica Fuller, Michael Opperman, Andrew Tollefson] Received: July 29, 2011

The Distribution of the Number of Missing Sums in Sumsets

Oleg Lazarev (olazarev@princeton.edu)
Williams College [Mentor: Steven Miller]

Abstract of Report Talk: For any finite set of integers $A$, define its sumset $A + A$ to be $\{x + y : x, y \in A\}$. Sumsets appear in many of the deepest questions of number theory, such as Goldbach’s problem and the Twin Prime Conjecture, and there is a lot of work devoted to understanding these structures.

In a recent paper, Martin & O’Bryant studied sum-dominant sets, where $|A + A| > |A - A|$. These are interesting as one expects a generic $A$ to have $|A + A| < |A - A|$ (as addition is commutative but subtraction is not). They prove a positive percentage of all sets are sum-dominant, and investigate the distribution of $|A + A|$ given the uniform distribution on subsets $A \subset \{0, 1, \ldots, n - 1\}$. They conjecture the existence of a limiting distribution for $|A + A|$ and show that the expectation of $|A + A|$ is $2n - 11 + O((3/4)^{n/2})$.

We prove exponential upper and lower bounds (independent of $n$) for $P(A \subset \{0, \ldots, n-1\} : |A + A| = 2n - 1 - k)$, in particular showing that all higher moments of $|A + A|$ are finite as $n \to \infty$. We also derive an explicit formula for the variance of $|A + A|$ in terms of Fibonacci numbers via a graph-theoretic approach.

[LO24231557]

Received: August 1, 2011
**ON AN EXTREMAL PROBLEM OF POLYA**

**Tuan N Le**

*California State Uni. of Fullerton*

[Mentor: Zair Ibragimov]

**Abstract of Report Talk:** The notion of transfinite diameter of planar sets was introduced by M. Fekete around the 1920’s. It plays an important role in classical complex analysis and is related to other well-known concepts such as the logarithmic capacity and Chebyshev polynomials. The transfinite diameter of a compact set in the complex plane is the limit of n-diameters of the set. For each \( n \geq 3 \), the n-diameter \( d_n(E) \) of \( E \) is given by

\[
d_n(E) = \max \left\{ \prod_{1 \leq i < j \leq n} |z_i - z_j|^{\frac{2}{n(n-1)}} \right\},
\]

where the maximum is taken over all n-tuples \( \{z_1, z_2, \ldots, z_n\} \) of points in \( E \).

The following is the extremal problem of G. Polya: among all n-tuples \( E = \{z_1, z_2, \ldots, z_n\} \) with \( |z_i| \leq 1 \), find one with the largest n-diameter. The solution of this problem, attributed to Polya, is given below.

\[
d_n(E) \leq n^{\frac{1}{n-1}},
\]

and the equality holds for n-tuples of equally spaced points on the boundary of the unit disc \( D \). While investigating the transfinite diameter of sets of constant width, Prof. Zair Ibragimov was led to the following stronger version of Polya’s problem: among all n-tuples \( E = \{z_1, z_2, \ldots, z_n\} \) with \( |z_i - z_j| \leq 2 (1 \leq i < j \leq n) \), find one with the largest n-diameter. He conjectured that the extremal configuration will also be the vertices of a regular n-gon, at least when \( n \) is odd. In this talk, we will show that this is indeed true in the case of 5-tuples and partially verify for 7-tuples, in which the vertices of the regular 5-gon and 7-gon both have the maximum 5-diameter and 7-diameter, respectively. Finally, we will also show that among all special 4-gon configurations with diameter 2 (e.g: square, rectangle, diamond, trapezoid and isosceles trapezoid), the trapezoid has the maximum 4-diameter.

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**THE n-LEVEL DENSITY OF ZEROS OF QUADRATIC DIRICHLET L-FUNCTIONS**

**Jake A. Levinson**

*Williams College*

[Mentor: Steven Miller]

**Abstract of Report Talk:** The statistical distributions of zeros of \( L \)-functions have wide-ranging applications in number theory and geometry. \( L \)-functions have been studied in connection with random matrix theory, which provides easier methods of computing these distributions. One statistic, the \( n \)-level density of low-lying zeros for a family of \( L \)-functions, measures the distribution of zeros near the central point \( s = 1/2 \). While all families of \( L \)-functions have the same zero statistics far away from the central point, near the central point family-dependent behavior emerges, governed by the arithmetic of the family.

According to the Density Conjecture of Katz and Sarnak, this statistic depends on a classical compact group associated to the family. We extend previous work by Gao, who computed the \( n \)-level densities of quadratic Dirichlet \( L \)-functions for suitably restricted test functions but, due to combinatorial obstructions, could only show equality with random matrix theory up to \( n = 3 \). We develop a new vantage point to bypass these obstructions. The key step is to find a ‘canonical’ form for several Fourier Transform identities, which allows us to prove them via combinatorial arguments. Our main results are to confirm up to \( n = 6 \) that, for test functions of suitable support, the density is that of symplectic matrices, and to conjecture that larger \( n \) follows from a certain identity.

Received: August 1, 2011
Fraction of Nonnegative Polynomials that are Sums of Squares

Caitlin A Lownes (clownes@mit.edu)
Texas A & M University [Mentor: Joseph Rojas]

Abstract of Summary Talk: Polynomials that are sums of squares (SOS) can be efficiently optimized via semidefinite programming. In this presentation, we investigate when a nonnegative polynomial \( p \in \mathbb{R}[x_1, \ldots, x_n] \) is SOS. For nonnegative polynomials of fixed degree, previous results by Blekherman show that, as \( n \to \infty \), the fraction of nonnegative polynomials that are SOS approaches zero. However, these bounds are loose, and this fraction is unknown for most polynomials of low degree in few variables. Our research focuses on estimating this fraction for nonnegative bivariate polynomials of degree at most four in each variable. The fraction of nonnegative polynomials that are SOS can be estimated as the ratio of volumes of two naturally definable convex bodies of dimension 24. To avoid computing these volumes directly, we implemented a version of Smith’s rapidly mixing hit and run technique for uniform sampling from a convex body.

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Combinatorics and topology of graphs on surfaces.

Jonathan R Michel (michel.82@osu.edu)
The Ohio State University [Mentor: Sergei Chmutov]

Abstract of Report Talk: With a graph embedded into a surface one can associate many topological invariants. Examples are the homology classes of various cycles in the graph in the first homology group of the surface; or the genera of small neighborhoods of various subgraphs of the graphs, etc. On other hand with such a graph one can also associate a lot of combinatorial invariants such as the cycle matroid and bond matroid of the graph and of the Poincare dual graph on the surface, etc. We show how these invariants are related to each other. For instance, the rank of the the image of a subgraph in the first homology group of the surface is equal to the nullity of the edges of this subgraph in the bond matroid of the dual graph; the difference between ranks of the bond matroid of the dual graph and the cycle matroid of the original graph is equal to twice the genus of the surface, and so on. Formally this leads to an expression of the matroidal Las Vergnas polynomial as a specialization of the topological Krushkal polynomial for a graph cellularly embedded into a surface. The details of this research are explained in a preprint arXiv:1012.5053v1[math.CO] which is to appear in Quantum Topology.

[Joint with Ross Askanazi, Patrick Stollenwerk]
Recursion in the BRWT Polynomial of Ribbon Graph Families

Murphy Kate L Montee
Louisiana State University

Abstract of Summary Talk: Given a nested tower of ribbon graphs successively built from a given ribbon graph pattern, we extend the transfer method of N. Biggs and Noy-Ribo for the computation of the Tutte polynomial to the computation of the Bollobas-Riordan-Whitney-Tutte rank polynomial of ribbon graphs. Based on work begun by Jordan Keller, the concept of an amalgamation state is non-trivially extended from the connected partitions of the set of amalgamating vertices to include cyclic partitions of sectors in the amalgamating vertices for ribbon graphs. The key result is the proof that changes in certain ribbon graph statistics (rank, nullity, genus) depend only on the amalgamation state of an edge subset. The characteristic polynomial of the state transition matrix of the transfer method yields a recursion between the rank polynomials of the ribbon graphs in a tower. Applications to knot theory are given to find recursions among the Jones polynomials of links whose diagrams form a tower using the specialization of the BRWT polynomial of the ribbon graph obtained from the all-A smoothing of a knot to the Jones polynomial.

The Mean and Variance of $\omega(n)$

Eric P Naslund
University of Montreal

Abstract of Report Talk: The function $\omega(n)$, the number of distinct prime factors of $n$, is chaotic and can be studied from a statistical viewpoint. The mean and variance were first looked at in 1917 by Hardy and Ramanujan. In 1940 Erdős and Kac were able to prove their famous theorem relating the distribution of $\omega(n)$ to the normal distribution by calculating higher moments. In 1975 and 1977, Persi Diaconis gave an asymptotic expansion for the mean and variance using complex analysis. My main result is a more precise asymptotic for the mean, as well as a simpler proof for the expansion obtained by Diaconis in 1977.

Achieving Minimal-Maximum In-Degree and Lexicographically Minimal In-Degree Sequence of a Digraph

Antonio Ochoa
Oregon State University

Abstract of Poster Presentation: We consider the problem of orienting an undirected graph so that the resulting orientation achieves minimal-maximum in-degree or a minimum of a convex function of the in-degrees. The minimum of a convex function of the in-degrees is realized by the lexicographically minimal in-degree sequence. In particular we also stipulate the resulting graph to be strongly connected. First we prove a lower bound for the minimal-maximum in-degree of a strongly connected directed graph. We then present a polynomial-time algorithm that achieves this lower bound. We also give a polynomial-time algorithm that orients a graph so that it achieves a strong connected orientation with a lexicographically minimal in-degree sequence.

[OA25154630]

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ON THE ATTAINABLE CONSECUTIVE ORDERS OF $\mathbb{Z}_n$

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University of California, Santa Barbara  
[Mentor: Maribel Bueno]

Abstract of Report Talk: Let $S \subseteq \mathbb{Z}_n$. For $k \in \mathbb{N}$ we denote by $kS$ the set $kS := \{a_1 + \cdots + a_k \mod n \mid a_i \in S\}$

$S$ is said to be a basis for $\mathbb{Z}_n$ if there exists $k \in \mathbb{Z}_n$ such that $kS = \mathbb{Z}_n$. The smallest such $k$ is said to be the order of $S$ denoted by $\text{order}(S)$. In our project we study the set $E_n = \{\text{order}(S) \mid S \subseteq \mathbb{Z}_n \} \subseteq \mathbb{Z}_n$. Determining the structure of $E_n$ has been an open problem since 1974.

It is well-known that $\{1, n-1\} \subseteq E_n$ but $E_n \not\subseteq \{1, \ldots, n-1\}$, that is, there are some gaps in $E_n$. An important question is to determine the largest integer $k$ such that $\{1, \ldots, k\} \subseteq E_n$. In a previous paper, it was conjectured that $k \leq n^{2/3}$. Recently, it was proven that $k \geq \lfloor \sqrt{n} \rfloor$. We have proven that, for every positive integer $n$, there exists $M_n \in [1, 1.3]$ such that the integers in $[1, n^{3/5}/(M_n + n^{-2/5})] + [\sqrt{n^{2/5}} - 1] - 1$ are attainable. In fact, we may choose $M_n$ so that $\{M_n\}$ is a decreasing sequence. Thus for large enough $n$, $[1, n^{3/5}]$ are attainable orders. Moreover, based on numerical experiments, we conjecture that this is an optimal bound.

To prove the previous result we constructed a family of structured bases whose order was computed explicitly. These bases provide us with valuable information about the whole distribution of orders in $E_n$.

This problem has equivalent statements in terms of Boolean circulant primitive matrices and circulant digraphs. Applications of this question appear in diverse areas such as Markov processes, coding theory, and quantum information.

[Joint with Magdalene Flaris]  
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REDUCED COLORED KOVANOV HOMOLOGY ON KNOTS

Lisa M Piccirillo (picciril@bc.edu)  
Boston College  
[Mentor: Elisenda Grigsby]

Abstract of Poster Presentation: Khovanov homology associates to a knot a bigraded chain complex whose homology is a knot invariant (i.e., independent of the particular diagram used to describe the knot). Moreover, Khovanov homology categorifies the classical Jones polynomial in the sense that the graded Euler characteristic of the Khovanov homology of a knot yields the Jones polynomial of the knot. It was proved by Manolescu and Ozsvath that the Khovanov homology of alternating (and, more generally, quasi-alternating) knots is homologically thin, hence determined by the Jones polynomial and the signature. In Categorifications of the Colored Jones Polynomial, Khovanov describes a generalization of his construction categorifying the reduced colored Jones polynomial. A natural question to consider is whether Khovanov’s reduced colored Khovanov homology is also homologically thin on alternating knots. We give a number of counterexamples to this question, obtained computationally using a Mathematica program we developed, building upon Dror Bar-Natans FastKh algorithm contained in the open source KnotTheory package.

[PL25145351]  
Received: July 28, 2011
**Presentation of the Motzkin Algebra**

Eliezer Posner  
*University of California Santa Barbara*  
(Mentor: Stephan Bigelow)

Abstract of Report Talk: In 2011, Halverson introduced the Motzkin algebra, a fascinating generalization of the Temperley-Lieb algebra, whose elements are diagrams that can be multiplied by stacking one on top of the other. Halverson gave a diagrammatic algorithm for decomposing any Motzkin diagram into diagrams of three subalgebras: the right planar rook algebra, the Temperley-Lieb algebra, and the left planar rook algebra. We first explored the right and left planar rook subalgebras, proving that their cardinalities are Catalan numbers. We found presentations for these algebras by generators and relations, using a counting argument to prove that our relations suffice. We then turned to the newly-developed Motzkin algebra, where we described Halverson’s decomposition algorithm algebraically and found a presentation by generators and relations using a counting argument but with a much more sophisticated algorithm.

[Joint with Kristofer Hatch, Megan Ly]  
Received: August 1, 2011

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**Random walks on barycentric subdivisions and the Strichartz hexacarpet**

Diwakar M Raisingh  
*University of Connecticut*  
(Mentor: Teplyaev Alexander)

Abstract of Report Talk: Building on previous work regarding repeated barycentric subdivision of simplexes, we compute a large amount of data and prove several conjectures. We investigate properties of the spectrum of the Laplacian on the 3-simplex that has undergone Barycentric subdivisions and on the self-similar fractals, the Strichartz hexacarpet and the octagasket, both of which we present. We determined the Laplacian under Neumann boundary conditions using the method of averages defined by Kusouka and Zhou. Using this method, we have a wealth of data on the eigenvalues and eigenfunctions of the Laplacian and the behavior of the heat kernel on these spaces. Given the 1-skeleton of the $k$-th barycentric subdivision of a 2-simplex, we consider the weak planar dual of this graph. We prove several theorems regarding the graph diameter. We also conjecture that if the edges of the graph are weighted so that the maximum distance between two points in the graph metric is constant, then the graph and its dual induce the same completion metric on the 2-simplex in the limit as $k$ goes to infinity.

[Joint with Matthew Begue, Daniel Kelleher]  
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THE JONES AND ALEXANDER POLYNOMIALS THROUGH REPRESENTATIONS OF ROOK ALGEBRAS

Eric G Ramos
Ren Yi
University of California Santa Barbara

Abstract of Poster Presentation: In the 1920’s, Artin defined the braid group, $B_n$, in an attempt to understand knots in a more algebraic setting. A braid is a certain arrangement of strings in three-dimensional space. It is a celebrated theorem of Alexander that every knot is obtainable from a braid by identifying the endpoints of each string. Because of this correspondence, the Jones and Alexander polynomials, two of the most important knot invariants, can be described completely using the braid group. There has been a recent growth of interest in other diagrammatic algebras, whose elements have a similar topological flavor to the braid group. These have wide ranging applications in areas including representation theory and quantum computation. We consider representations of the braid group when passed through another diagrammatic algebra, the planar rook algebra. By studying traces of these matrices, we recover both the Jones and Alexander polynomials.

GRAPH THEORETIC APPROACH IN MATRIX COMPLETION PROBLEMS

Carissa E Romero
Cal State, Channel Islands

Abstract of Poster Presentation: A matrix completion problem involves completing a partially specified matrix to satisfy a given property. The focus of this presentation is completing the partially specified matrix so that it will commute with a fully specified matrix. In particular, given a fully specified matrix $A$, and a partially specified matrix $X$, when can we complete the remaining entries in $X$ so that the equation $AX -XA = 0$ will be satisfied? There are three approaches used to complete matrices, one of which is the Graph Theoretic Approach, which is also the focus of this presentation. The main theorem classifies all admissible patterns for a Jordan Block. This allows us to identify all patterns in a partially specified matrix $X$ so that $X$ can be completed to commute with a Jordan Block. The Classification Theorem is also extended to matrices with multiple Jordan Blocks and matrices that are permutation similar to a Jordan Block.

INVARIANT MEASURES FOR HYBRID STOCHASTIC SYSTEMS

Anthony Sanchez
Iowa State University

Abstract of Poster Presentation: Dynamical systems give one the ability to analyze the way systems evolve through time. Usually these are differential equations that model real world phenomena. Unfortunately, these models are limited in the sense that they cannot account for random events that may occur like friction or wind resistance. However, these random developments can often be modeled with Markov chains and processes. By uniting the two models one can see how these dynamical systems behave with the perturbation induced by Markov processes, but in doing so create a hybrid system where one now must simultaneously study the dynamical system and Markov process. I begin by examining the limit sets of these hybrid systems and what happens as they approach the limit sets. In particular, I analyze not just limit sets, but also prove the existence of invariant measures for these hybrid systems, as well as supply concrete examples with visuals that provide insight to the behavior of these system.
Minimal Pentagonal Tilings

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Luis A Sordo Vieira (dv8603@wayne.edu)
Williams College [Mentor: Frank Morgan]

Abstract of Report Talk: In 2001, Thomas Hales proved that regular hexagons provide a least-perimeter unit-area tiling of the plane, better for example than squares and equilateral triangles, which are minimizing for polygons with three or four sides respectively, and no worse than a mixture of any other shapes.

We seek the least-perimeter unit-area tiling of the plane by pentagons. Unfortunately, regular pentagons do not tile. Work by Frank Morgan and students resulted in a proof that two other pentagons, called Cairo and Prismatic, yield least-perimeter unit-area tilings by convex pentagons. The original version of the paper asked whether there exist tilings by mixtures of these two pentagons. We have found uncountably many such mixtures and classified the doubly periodic ones by their wallpaper symmetry groups.

We also consider tilings by mixtures of convex and nonconvex pentagons and perimeter-minimizing tilings on various flat tori. It is unknown whether a mixture of nonconvex and convex pentagons is better than the Cairo and Prismatic tilings in the plane. However, we establish a bound on the ratio of nonconvex to convex pentagons. We also place a lower bound on the perimeter of a nonconvex unit-area pentagon given the reflex angle and its adjacent edges, and use this to prove that the unique perimeter-minimizing pentagonal tiling of the square torus of area 4 is by Cairo pentagons.

[Joint with Ping Ngai Chung, Yifei Li, Miguel Fernandez, Michael Mara, Isamar Rosa Plata, Elena Wikner] Received: August 1, 2011
Abstract of Poster Presentation:

A loopless directed graph $\Gamma$ with $v$ vertices is called a directed strongly regular graph with parameters $(v, k, t, \lambda, \mu)$ if and only if $\Gamma$ satisfies the following conditions: (i) every vertex has in-degree and out-degree $k$, (ii) every vertex $x$ has $t$ out-neighbors, all of which are also in-neighbors of $x$, and (iii) the number of directed paths of length two from a vertex $x$ to another vertex $y$ is $\lambda$ if there is an edge from $x$ to $y$, and is $\mu$ if there is no edge from $x$ to $y$.

We have been investigating various tactical configurations that produce DSRGs and characterizing those producing DSRGs with certain parameter sets. A tactical configuration with parameters $(v, b, k, r)$ is a finite point-line incidence structure with $v$ points and $b$ lines (or blocks) such that each line is incident with $k$ points and each point is incident with $r$ lines. Infinite families of DSRGs are obtained from flags (incident point-line pairs) or antiflags of certain tactical configurations. As a major accomplishment we have proved the following fact.

Finding 1. The necessary and sufficient conditions for a tactical configuration to yield such DSRGs are the following properties: “For every point $x \in P$ and every block $B \in \mathcal{B}$ in the tactical configuration, the number $\alpha(x, B)$ of flags $(y, C) \in \mathcal{I}$ such that $y \in B \setminus \{x\}$, $C \in x$ and $C \neq B$, depends only on the incidence between $x$ and $B$.”

It is known that all strongly regular graphs have large automorphism groups that are vertex transitive. However, the automorphism groups of DSRGs are usually small and non-transitive. In the efforts of finding DSRGs that have large ‘symmetry’, we find an infinite class of DSRGs that has vertex-transitive automorphism groups. Our construction uses finite fields of (odd) prime order and their cosets of multiplicative subgroups. Although this class of DSRGs were previously known, our construction method is different, and hence we can compute their automorphisms by using the properties of underlying finite fields. We briefly summarize our findings here.

Let $\ell, s$ be integers greater than or equal to 2 such that $\ell s + 1$ is prime. Let $Q$ be the finite field of order $q = \ell s + 1$, and let $g$ be a primitive element of $Q$. Let $H$ be the subgroup generated by $g^\ell$ of $Q^*$ with index $\ell$. For each $i \in Q$, define a partition of $Q \setminus \{i\}$ into $s$ parts of size $\ell$. Let

$$Q \setminus \{i\} = \bigcup_{j=0}^{s-1} B_{ij} \text{ where } B_{ij} = i + g^j H \pmod{\ell s + 1}, \quad \text{and } \mathcal{B} = \bigcup_{i=0}^{q} \bigcup_{j=0}^{s-1} B_{ij}.$$ 

Then the pair $T = (Q, \mathcal{B})$ forms a tactical configuration with parameters $(v, b, k, r) = (\ell s + 1, s(\ell s + 1), \ell, \ell s)$ under natural point-block incidence.
Finding 2. Let $\Gamma = \Gamma(\mathcal{T})$ be the directed graph defined by

$$V(\Gamma) = \{(p, B) \in P \times B : p \notin B\}$$

and adjacency by

$$(p, B) \to (q, C) \text{ if and only if } p \in C.$$ 

Then the graph $\Gamma$ is a DSRG with parameters $(s(\ell s + 1), \ell s, \ell, \ell - 1, \ell)$.

Finding 3. The full automorphism group of the DSRG $(s(\ell s + 1), \ell s, \ell, \ell - 1, \ell)$ constructed as above acts transitively on its vertex set and its rank is $s(s+1)$.

Finding 4. For $s = 2$, the rank 6 transitive automorphism group of the DSRG $(2(2\ell + 1), 2\ell, \ell, \ell - 1, \ell)$ gives rise to a 5-class non-commutative association scheme whose 2-class fusion scheme yields a pair of strongly regular graphs.

Finding 5. For $\ell = 2$, the vertex-transitive automorphism group of the DSRG $(s(2s + 1), 2s, 2, 1, 2)$ gives rise to a $s(s+1)-1$-class non-commutative association scheme whose 2-class fusion scheme is isomorphic to Johnson scheme $J(2s + 1, 2)$.

[SG18195840]
[Joint with Oktay Olmez, Charles Watts, Angelica Gonzalez]
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ACHIEVING ALL RADIO NUMBERS
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Abstract of Report Talk: For a connected graph $G$, a radio labeling is a function $c : V(G) \to \mathbb{Z}^+$ such that for every pair of vertices $u, v$ in $V(G)$, the radio condition is satisfied:

$$\text{distance}(u,v) + |c(u) - c(v)| \geq \text{diameter}(G) + 1.$$ 

The span of a radio labeling $c$ is the largest integer in the image of $c$. The radio number of a graph $G$ is the smallest integer $M$ such that $\text{span}(c) = M$ for some radio labeling $c$. It is known that a graph of $n$ vertices has a radio number of at least $n$ and at most $(n - 1)/2 + r$, where $r$ is determined by the parity of $n$. This paper defines and examines three-parameter graphs known as Sok graphs. We show that for all but one integer between the known minimum and maximum, there exists a Sok graph whose radio number is that integer. Further we analyze a lower bound technique known as distance maximization with the goal of characterizing graphs for which the calculated lower bound is the radio number. The results of this work entirely settle the question of what the possible radio numbers are for graphs of order $n$. [SB25150313]

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SLOWLY GROWING BACKWARDS ORBITS OF POINTS UNDER A RATIONAL FUNCTION

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University of Michigan [Mentor: Michael Zieve]

Abstract of Report Talk: Let \( f(z) \in \mathbb{C}(z) \) be a degree-\( d \) rational function with complex coefficients, and let \( \phi: \mathbb{S}^1 \to \mathbb{S}^1 \) be the map \( c \mapsto f(c) \) from the Riemann sphere to itself. We show that, if \( d > 1 \) and \( r > 3 \), then any \( P \in \mathbb{S}^1 \) for which \( \#\phi^{-r}(P) > 1 \) must in fact satisfy \( \#\phi^{-r}(P) \geq \left\lceil \frac{d}{d^2 + d + 1} \right\rceil \). Conversely, we describe all functions for which equality holds; such functions exist for each \( d \) and \( r \). The proofs involve the Riemann–Hurwitz genus formula and a study of the ramification of \( \phi \) and its iterates.

This result improves previous results of Silverman (Duke Math. J., 1993) and Faber–Granville (J. Reine Angew. Math., 2011). As a corollary, we show that if \( f(z) \in \mathbb{Q}(z) \) has degree \( d > 2 \) and \( f(f(z)) \) is not a polynomial, then there are only finitely many rational numbers \( c \) for which \( f(f(f(c))) \) is an integer.

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THE DELTA SQUARED PROCESS AND THE FOURIER SERIES OF FUNCTIONS WITH MULTIPLE JUMPS

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Abstract of Poster Presentation: For a function \( f \) on \([−\pi, \pi]\), we define \( \hat{f}(k) := \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)e^{-ikx}dx \) and set \( S_nf(x) := \sum_{k=-n}^{n} \hat{f}(k)e^{ikx} \). If \( f \) is square integrable, then \( S_nf \to f \) in \( L^2 \), and numerous theorems give conditions under which \( S_nf(x) \to f(x) \). In practice, however, this convergence can be quite slow, and it is useful to find algorithms which can accelerate it. There are many algorithms to accelerate the convergence of numerical sequences and the question arises as to whether these can be used to accelerate the convergence of the sequence \( \{S_nf(x)\} \). In this talk we consider a well-known sequence acceleration method -the \( \delta^2 \) process- and investigate its effect on the sequence \( \{S_nf(x)\} \). In particular, we consider piecewise smooth functions with a finite number of jump discontinuities, which are known to have Fourier series which converge slowly. We prove that in most cases the application of this transform not only fails to accelerate convergence but actually destroys it.

[Joint with Emily Jennings, Charles Moore, Daniel Muniz]

Received: August 1, 2011
Generic polynomials for $p$–groups in characteristic $p$

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[Mentor: Jorge Morales]

Abstract of Report Talk: Generic polynomials are a constructive approach to the inverse Galois problem in that they parametrize all Galois extensions with a base field containing a given field $K$ and with a given Galois group $G$. In particular, if $K = \mathbb{F}_p$, then a generic polynomial would parametrize all $G$–extensions of any base field of characteristic $p$. We apply invariant theory and adapt the theory of Frobenius modules to the problem of finding generic polynomials for $p$–groups in characteristic $p$. In particular, we include explicit computations of generic polynomials in characteristic 2 for all groups of order 16 that cannot be expressed as a direct product of proper subgroups, some groups of order 32, and some 3–groups in characteristic 3.

Also, if $K$ is a field, we can define the action of a matrix group $G$ of $m$ by $m$ matrices on $K(x_1, \ldots, x_m)$ by fixing a basis $x_1, \ldots, x_m$ and extending the action of $G$ on an $m$–dimensional vector space over $K$ to the fraction field of the symmetric algebra. Miyata has shown that, for $G$ consisting of $m$ by $m$ upper triangular matrices, $K(x_1, \ldots, x_m)^G$ is purely transcendental over $K$ with $K(x_1, \ldots, x_m)^G = K(\phi_1, \ldots, \phi_m)$ provided that $\phi_i \in K(x_1, \ldots, x_{i-1})[x_i]^G$ is of minimal positive degree in $x_i$. For all finite groups $G$ consisting of upper triangular matrices, we prove a result that determines the degree of each invariant $\phi_i$ in $x_i$. We present some corollaries to this result, and apply it to the computation of generic polynomials for $p$–groups in characteristic $p$ through invariant theory.

Received: July 28, 2011

Generalized Roundness of Graphs

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[Mentor: Anthony Weston]

Abstract of Poster Presentation: Stephen Sanchez (2011) provides a method to calculate the maximal generalized roundness of finite metric spaces. In this paper we use his method to compute the generalized roundness of all windmill graphs endowed with the usual graph metric. We also use techniques from Wolf (2010) to show that the strict 1-negative gap of all finite triangulated cycle graphs is positive, thus showing all finite triangulated cycles have a generalized roundness strictly greater than one. This work was completed at the 2011 Cornell University Summer Mathematics Institute.

[Joint with Toyin Alli, Mathav Murugan, Jenny Peterson, Elizabeth Wesson]

Received: August 1, 2011
STARS AND FAREYS: A SCREEN SIZE ROMANCE

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Abstract of Report Talk: The screen size of the graph $K_{1,n-1}$ (i.e. a star of order $n$) is the smallest number such that the $k \times k$ integer lattice supports a drawing of $K_{1,n-1}$, where each vertex is a lattice point and the edges are drawn as non-overlapping line segments. We find that the screen size of a star is related to the number of visible points on an integer lattice (i.e. the screen). In particular, given a screen of odd size $s = 2k + 1$, $k \in \mathbb{N}$, if $K_{1,n-1}$ is centered in the middle of the screen, then it can have at most $8F\left(\frac{s-1}{2}\right) + 9$ vertices, where $F(i)$, called the Farey number of $i$, is the length of the Farey sequence for $i$.

However, the center of the screen is not necessarily the point that will fit the largest possible star. Thus, we derive a 2-dimensional generalization of Farey numbers that gives the exact number of visible points from any point of an integer lattice by looking at the number of co-prime pairs in a rectangle and applying the sieve principle. We then generalize Euler’s totient function in order to investigate the asymptotic properties and provide bounds for our new function.

[Joint with Charles S. Berahas and Jian Shen]
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ON SOME MULTICOLOR RAMSEY NUMBERS INVOLVING $K_3 + e$ AND $K_4 - e$

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Rochester Inst. of Technology  [Mentor: Stanislaw Radziszowski]

Abstract of Report Talk: The Ramsey number $R(G_1, G_2, G_3)$ is the smallest $n$ such that for all 3-colorings of the edges of $K_n$ there is a monochromatic $G_1$ in the first color, $G_2$ in the second color, or $G_3$ in the third color. We study the bounds on various 3-color Ramsey numbers $R(G_1, G_2, G_3)$, where $G_i \in \{K_3, K_3 + e, K_4 - e, K_4\}$. The minimal and maximal combinations of $G_i$’s correspond to the classical Ramsey numbers $R_3(K_3)$ and $R_3(K_4)$ respectively, where $R_3(G) = R(G, G, G)$. Here, we focus on the much less studied combinations between these two cases.

Through computational and theoretical means we establish that $R(K_3, K_3, K_4 - e) = 17$, and by construction we raise the lower bounds on $R(K_3, K_4 - e, K_4 - e)$ and $R(K_4, K_4 - e, K_4 - e)$. For some $G$ and $H$ it was known that $R(K_3, G, H) = R(K_3 + e, G, H)$; we prove this is true for several more cases including $R(K_3, K_3, K_4 - e) = R(K_3 + e, K_3 + e, K_4 - e)$.

Ramsey numbers generalize to more colors, such as in the famous 4-color case of $R_4(K_3)$, where monochromatic triangles are avoided. It is known that $51 \leq R_4(K_3) \leq 62$. We prove the surprising theorem stating that if $R_4(K_3) = 51$ then $R_4(K_3 + e) = 52$, otherwise $R_4(K_3 + e) = R_4(K_3)$.

[Joint with Daniel Shane Shetler, Whitworth University, Spokane, WA]
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RATIONAL LINEAR SPACES ON HYPERSURFACES OVER QUASIALGEBRAICALLY CLOSED FIELDS

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[Mentor: Craig Spencer]

Abstract of Report Talk: A field \( k \) is called a \( C_i \) field if every form of degree \( d \) with coefficients in \( k \) having more than \( d^i \) variables has a non-trivial zero. \( C_i \) theory is a powerful tool in that it provides a sharp bound for the number of variables necessary for an arbitrary form to have a non-trivial zero. Suppose \( f_1, f_2, \ldots, f_r \) are forms over \( k \) of degree \( d_1, d_2, \ldots, d_r \), respectively. Using extensive combinatorial techniques and \( C_i \) theory, we determine a bound for the number of variables necessary for \( f_1, \ldots, f_r \) to have a projective \( l \)-dimensional linear space of simultaneous zeroes, in terms of \( l, d_1, d_2, \ldots, d_r \). The bound we obtain when \( k = \mathbb{F}_q(t) \), a \( C_2 \)-field, is much stronger than the analogous bound for a system of odd-degree forms over \( \mathbb{Q} \). We also present an application involving monic irreducible polynomials in \( \mathbb{F}_q(t) \).

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LOW-LYING ZEROS OF CUSPIDAL MAASS FORMS

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[Mentor: Steven Miller]

Abstract of Report Talk: Though Random Matrix Theory was developed to explain the energy levels of heavy nuclei, later it was observed that similar answers are found for zeros of \( L \)-functions, and since then RMT has modeled their behavior. These zeros are connected to many problems in number theory, from the prime number theorem to the class number problem. The Katz-Sarnak Density Conjecture states that the behavior of zeros of a family of \( L \)-functions near the central point (as the conductors tend to zero) agree with the behavior of eigenvalues near 1 of a classical compact group (as the matrix size tends to infinity).

Maass forms are smooth functions on the upper half plane, are invariant under the action of \( \text{SL}_2(\mathbb{Z}) \), are eigenfunctions of the non-Euclidean Laplacian, and are a natural generalization of the Riemann zeta function. While they arise in a variety of problems in number theory, they are significantly harder to work with than their cousins (the holomorphic cusp forms) as the averaging formula here is significantly more unwieldy. We study the distribution of zeros near the central point of \( L \)-functions of level 1 Maass forms; this is essentially summing a smooth test function whose Fourier transform is compactly supported over the scaled zeros.

Using the Petersson formula, Iwaniec, Luo and Sarnak proved that the zeros near the central point of holomorphic cusp forms agree with the eigenvalues of orthogonal matrices for suitably restricted test functions. We prove a similar result for Maass forms. We derive an explicit formula (via complex analysis) relating sums of our test function at scaled zeros to sums of the Fourier transform at the primes weighted by the Maass form’s coefficients, and use the Kuznetsov trace formula to average over the family. There are numerous technical obstructions in handling the terms in the trace formula, which are surmounted through the use of smooth weight functions and results on Kloosterman sums and Bessel and hyperbolic functions.

[Joint with Nadine Amersi, Geoff Iyer]

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