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Abstracts of Presentations

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Plenary Lectures

BLOWN AWAY: WHAT KNOT TO DO WHEN SAILING

Colin Adams
Williams College

Abstract of Lecture:
by Sir Randolph Bacon III cousin-in-law to Colin Adams, Williams College.
Being a tale of adventure on the high seas involving great risk to the tale teller, and how an understanding of the mathematical theory of knots saved his bacon. No particular nautical or mathematical background assumed.

FROM COMBINATORICS TO DYNAMICS AND BACK

Bryna Kra
Northwestern University

Abstract of Lecture: Ramsey Theory is the study of phenomenon that for certain large structures, a 'large' portion of the structure must contain a smaller scale substructure of the same type. 'Large' can have many interpretations, for example, in terms of density of a set of integers. One of the earliest results in this area is van der Waerden's Theorem: if the integers are colored using finitely many colors, then there are monochromatic arithmetic progressions of arbitrary length. Such properties of sets can be naturally studied in terms of dynamical systems, and this approach has lead to numerous breakthroughs. I will give an overview of these interactions between dynamical systems and problems in additive combinatorics.

COMBINATORICS OF STAIRCASE TABLEAUX

Lauren Williams
University of California, Berkeley

Abstract of Lecture: I will give an introduction to some new combinatorial objects called staircase tableaux. I will begin by describing some of their nice enumerative properties – various subsets of them have cardinality $n!$, $(2n+1)!!$, Fibonacci numbers, etc. – and then explain how these tableaux can help us solve a problem about traffic flow.

Student Presentations

(Alphabetically by last name of primary presenter)

(See also the *Student Index* at the end of this booklet)

RICCI FLOW ON WARPED PRODUCT MANIFOLDS AND PINCHING ESTIMATES

Robert J. Abramovic (rabramo1@jhu.edu)
Cornell University [Mentor:Xiaodong Cao]

Abstract of Report Talk: We introduce Ricci flow on a warped product space $M \times S^1$, where M is a closed, i.e. compact without boundary, Riemannian manifold, via a smooth positive function $f : M \to \mathbb{R}$. Considering the evolution equation of a Harnack-type quantity and assuming long time existence of the flow, we prove that, when M is a surface, the metric will approach the associated direct product. Moving to the more general case, we prove via a pinching estimate that the Ricci curvature tensor restricted to the S^1 direction is always bounded from below, comparing it to the total scalar curvature. It is well-known that the Ricci curvature of a 3-dimensional manifold after dilation is always nonnegative, but this is not generally the case in four and higher dimensions. This result therefore has geometric significance when $M \times S^1$ is of dimension 4 and higher. Bounding the Ricci curvature from below extracts geometric and topological information via comparison to a manifold whose Ricci curvature has a lower bound. If M is a 3-dimensional manifold, we consider the evolution of the Hessian with the eventual hope of bounding the Ricci curvature restricted to M from below as well. **[AR02154728]**

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A SIMULTANEOUS RANDOM WALK GAME

Jorge H. Banuelos (jbanuelo@macalester.edu)
Macalester College [Mentor:Andrew Beveridge]

Abstract of Summary Talk: Two tokens are placed on vertices of a tree graph. At each time step, one token is chosen and moved to a random neighboring point. In previous work, Tetali and Winkler studied the Angel strategy for bringing the tokens together as quickly as possible (on average), and the Demon strategy for delaying their collision as long as possible (on average). We build on these results by studying a game version of this process.

In our game, two players take turns choosing the token to move. Angel player hopes to bring the tokens together while Demon player tries to keep them apart. We present optimal strategies for both players on stars, paths, star-like graphs and m-ary trees. Our proofs employ couplings of random walks as well as strategy stealing arguments. **[BJ30124047]**

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BITING BOOLEAN FUNCTIONS

Maxwell L. Bileschi (mlbileschi@gmail.com)
Daniel Padgett (dpadgett@buffalo.edu)
University at Buffalo [Mentor:Thomas Cusick]

Abstract of Report Talk: This paper studies degree 3 Boolean functions of n variables $x_1, \dots, x_n$ which are rotation symmetric, that is, invariant under any cyclic shift of the indices of the variables. These rotation symmetric functions have been extensively studied in the last ten years or so because of their importance in cryptography. Some of the cryptographic applications are described in a 2002 paper of Cusick and Stănică. That paper gave a recursion for the truth tables and a nonhomogenous recursion for the (Hamming) weights of the homogeneous cubic rotation symmetric functions of n variables generated by the monomial $x_1x_2x_3$. Until now, this was the only investigation of the recursive structure of such functions generated by a single monomial. Here we provide an algorithm for finding a recursion for the truth tables for *any* such cubic rotation symmetric Boolean functions, as well as a *homogenous* recursion for their weights; in doing so we reduce the computation of a problem that is exponential in the number of variables to a problem that is exponential in the distance between the indices of a generating monomial. An intricate proof using linear algebra applied to characteristic matrices of the functions shows that the algorithm will always succeed. **[BM09155144]**

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EIGENVALUE DENSITY AND LEVEL-SPACING FOR SELF-ADJOINT NON-SYMMETRIC RANDOM MATRICES

Bill Karr (wkarr@iupui.edu)
Indiana University-Purdue University Indianapolis [Mentor:Yogesh Joglekar]

Abstract of Report Talk: Eugene Wigner's robust "semicircle law" for the eigenvalue density and his conjecture for the eigenvalue spacing statistics of random matrices laid the groundwork for random matrix theory. For a real symmetric $N \times N$ matrix whose entries are drawn from the same probability distribution $q(x)$ with zero mean and finite moments, the semicircle law states that as $N \to \infty$, the eigenvalue distribution (after a proper scaling that is independent of $q(x)$) is given by $p_W(\lambda) = 2\sqrt{1-\lambda^2}/\pi$, independent of the underlying distribution $q(x)$.

We investigate the eigenvalue density and level-spacing statistics of non-symmetric matrices that are self-adjoint with respect to a general inner-product, and thus have purely real spectra. For an inner-product on $\mathbb{R}^N$ defined by $(u, v)_f = \sum_{i=1}^{N} f(i)u_iv_i$, the self-adjoint matrices can be characterized by $M_{ij} = H_{ij}f(j)$ where the random matrix H satisfies the criteria for the Wigner's law. We present numerical results for the eigenvalue density distribution $p_{f,M}(x)$ of such $N \times N$ random matrices M. Combining them with analytical treatment, we show that as $N \to \infty$, the distribution is solely determined by the inner product f, and not dependent upon the underlying probability distribution used to obtain the random matrix M. We conclude by presenting numerical results for the eigenvalue level-spacing for such matrices. Our results provide a generalization of Wigner's law and conjecture that is applicable to self-adjoint, but non-symmetric, random matrices. **[BK02005217]**

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ON UNIVERSAL CYCLES FOR NEW CLASSES OF COMBINATORIAL STRUCTURES

Antonio Blanca (ablanca3@gatech.edu)
East Tennessee State University [Mentor:Anant Godbole]

Abstract of Poster Presentation: A universal cycle (u-cycle) is a compact listing of a collection of combinatorial objects. A classic example is the de Bruijn cycle of order n. Universal cycles have been analyzed with varying success for collections of subsets, multisets, partitions, permutations, discrete functions, labeled graphs, and other objects. We use natural encodings of combinatorial objects to show the existence of u-cycles for collections of subsets, matroids, restricted multisets, chains of subsets, multichains, and lattice paths. For subsets, we show that a u-cycle exists for the k-subsets of an n-set if we let k vary in a non zero length interval. We use this result to construct a "covering" of length $(1+o(1))\binom{n}{k}$ for all subsets of $[n]$ of size exactly k with a specific formula for the $o(1)$ term. We also show that u-cycles exist for all n-length words over some alphabet Σ, which contain all characters from $R \subset \Sigma$. Using this result we provide u-cycles for encodings of Sperner families of size 2 and proper chains of subsets. **[BA02114829]**

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AN ARROW GENERALIZATION OF THE BOLLOBAS-RIORDAN POLYNOMIAL

Clark W. Butler (butler.552@buckeyemail.osu.edu)
Ohio State University [Mentor:Sergei Chmutov]

Abstract of Report Talk: A virtual Thistlethwaite's theorem relates the Jones polynomial of a virtual link to the Bollobas-Riordan polynomial of a special ribbon graph. Recently, H. Dye and L. Kauffman discovered a strong extension of the Jones polynomial for virtual links, the arrow polynomial, which gives a lower bound on the virtual crossing number of a planar representation of the link. We create an additional arrow structure on ribbon graphs which allows us to extend the Bollobas-Riordan polynomial to ribbon graphs with this structure. Thus, we get a generalization of Thistlethwaite's theorem to the arrow polynomial of virtual links. This arrow extension of the Bollobas-Riordan polynomial possesses interesting combinatorial properties such as contraction-deletion and duality relations with respect to the set of edges of the ribbon graph. These relations generalize the corresponding properties of the Bollobas-Riordan and Tutte polynomials. **[BC01183340]**

[Joint with Rob Bradford]

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SMOOTH REPRESENTATIONS OF A P-ADIC HEISENBERG GROUP VIA SHEAVES ON THE DUAL SPACE

Justin Campbell (cjustinc@umich.edu)
University of Michigan [Mentor:Mitya Boyarchenko]

Abstract of Report Talk: François Rodier proved that it is possible to view smooth representations of certain totally disconnected abelian groups (the underlying abelian group of a finite-dimensional p-adic vector space, for example) as sheaves on the Pontryagin dual group. There is a suitable notion of dual space for non-abelian totally disconnected groups, but it necessarily includes representations which are not one-dimensional, and does not carry a group structure. Its so-called Fell topology is notoriously hard to work with, so we provide a characterization of this topology for a large class of totally disconnected groups (which includes, for example, p-adic unipotent groups) and use it to show that smooth representations still behave very much like sheaves in this generality. In particular, we give a sheaf-theoretic description of the category of smooth representations of a p-adic Heisenberg group. In the future we hope to use these methods, which might loosely be called geometric, to similarly describe smooth representations of any p-adic unipotent group. **[CJ28183805]**

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VIRTUAL MILNOR NUMBERS AND THE CONWAY POLYNOMIAL

Zhanpeng Cheng (cheng.309@osu.edu)
Ohio State University [Mentor:Sergei Chmutov]

Abstract of Report Talk: A well known theorem of T. Cochran states that the first nontrivial coefficient of the Conway polynomial for an algebraically split, 3-component classical link is equal to the square of the link's Milnor number. J. Levine and L. Traldi generalized this to an arbitrary number of components. We further generalize the result to virtual links. In this situation, we use a Gauss diagrammatic formula for the Milnor numbers obtained by M. Polyak and Jaeger's state model for the Conway polynomial described by S. Chmutov, M. Khoury, and A. Rossi. In addition, we present a free JavaScript program for computing Conway polynomials, Milnor numbers, and linking numbers of links. **[CZ01014435]**

[Joint with Jeff Lindquist, Theodore Dokos]

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STICK NUMBER OF KNOTS IN THE CUBIC LATTICE

Michelle Chu (mchu@emory.edu)
Stephanie A. Jensen (saj1@williams.edu)
Williams College [Mentor:Colin Adams]

Abstract of Report Talk: The stick number of knots, defined as the least number of straight sticks glued end-to-end needed to construct the knot, is a long-studied invariant of great interest to synthetic chemists. We now study the stick number of knots in the cubic lattice. Previous to this work, cubic lattice stick number was known only for the trefoil, the figure-eight knot, and 9_{47}. We use bridge index and construction to prove the lattice stick number of the infinite class of $(p, p+1)$-torus knots as well as of three other prime knots. Additionally, we find the lattice stick number for their compositions, cable satellites, and several links. Finally, we present linear upper and lower bounds in terms of crossing number. No background assumed. **[CM02112042]**

[Joint with Thomas Crawford, Kyler Siegel, and Liyang Zhang]

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A WINNING STRATEGY FOR TIC-TAC-TOE ON AN AFFINE PLANE OF ORDER 4

Matthew P. Conlen (mpconlen@umich.edu)
Juraj Milcak (j.milcak@utoronto.ca)
The Fields Institute [Mentor:Brett Stevens]

Abstract of Report Talk: Our research looks at the classic game Tic-Tac-Toe as played on finite geometries, in particular the affine plane of order 4. J. Yazinski and A. Insogna have given a computational proof that this game may always be won by the first player. We provide a simple winning strategy and proof of its correctness. To do so, we use mutually orthogonal latin squares to coordinatize points and place lines into parallel classes. Consider any subgraph of the game board such that the subgraph is itself an affine plane of order two. We denote the set of points of any such subgraph as $\mathcal{S}$. We prove that the first player is always able to create a set $\mathcal{S}$, and then show that this set can be extended to produce a winning line. **[CM02152935]**

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SPECTRAL ANALYSIS OF NON-HERMITIAN MATRICES

Amalia V. Culiuc (culiu22a@mtholyoke.edu)
Williams College [Mentor:Mihai Stoiciu]

Abstract of Report Talk: Motivated by work of Contedini-Embree-Trefethen and Goldsheid-Khoruzhenko, we investigate the spectral properties of certain classes of non-Hermitian matrices. We give parametrizations for curves in the plane that contain the spectrum of bi-diagonal matrices with periodic diagonal entries. In the case of period two, we find an asymptotic formula for the spacing between these eigenvalues.

We also study the pseudospectrum $\sigma_\varepsilon(A)$ of a general square matrix A. We generalize the Bauer–Fike Theorem and give lower and upper bounds to show that the asymptotic decay (as $\varepsilon \to 0$) of the diameter of $\sigma_\varepsilon(A)$ near the eigenvalue λ is of order $\varepsilon^{1/k}$, where k is the dimension of the largest Jordan block associated to λ. **[CA02140512]**

[Joint with Matthew Coudron, Philip Vu, Stephen Webster]

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ARITHMETIC PROGRESSIONS OVER QUADRATIC FIELDS

Alexander Diaz (alexander.diaz1@upr.edu)
Mathematical Sciences Research Institute [Mentor:Edray Goins]

Abstract of Poster Presentation: In 1640 Pierre De Fermat proposed to Bernard Frenicle de Bessy the problem of showing that there is no non-constant arithmetic progression of four squares over $\mathbb{Q}$. The proof of this proposition was published posthumously in 1780 by Leonhard Euler. However, Euler's proof does not hold over $\mathbb{Q}(\sqrt{D})$ where D is a squarefree integer. Arithmetic progressions of four squares over $\mathbb{Q}(\sqrt{D})$ correspond to rational points on the elliptic curve $E : y^2 = x^3 + 5x^2 + 4x$ and this is related to the quadratic twist $E^{(D)} : y^2 = x^3 + 5Dx^2 + 4D^2x$. We use methods of elliptic curves to discuss the existence of arithmetic progressions of squares over $\mathbb{Q}(\sqrt{D})$. **[DA19145437]**

[Joint with Markus Vasquez, Zach Flores]

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QUADRATIC FIELDS WITH CYCLIC 2-CLASS GROUPS

Carlos R. Dominguez (crd1@williams.edu)
Williams College [Mentor:Steven Miller]

Abstract of Report Talk: The class group is one of the most studied objects in algebra. Knowing its makeup is important as it has a multitude of applications, including playing a central role in Dirichlet's proof of the infinitude of primes in arithmetic progressions. Unfortunately, very little is known about its structure in general. We prove that there are infinitely many complex quadratic fields with cyclic 2-class groups of arbitrarily large order. Furthermore, for any integer $k > 1$, we construct quadratic fields with cyclic 2-class group of order exactly 2^k. In the proof we generalize analytic arguments by Perelli used to study the distribution of Goldbach numbers which can be represented by polynomials. Specifically, we extend the circle method to allow the incorporation of constraints forcing the primes to lie in specific congruence classes. These results allow us to exactly construct the desired quadratic fields. **[DC29134201]**

[Joint with Siman Wong]

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HYPERCYCLICITY AND SUBSPACE HYPERCYCLICITY OF INVERTIBLE OPERATORS IN HILBERT SPACE

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SUNY Potsdam-Clarkson [Mentor:Blair Madore]

Abstract of Report Talk: An operator T on a Hilbert space $\mathcal{H}$ is said to be hypercyclic if there exists a vector whose orbit under T is dense in $\mathcal{H}$. Surprising in its demonstration of linear chaos, hypercyclicity also has the potential to provide a solution to the invariant subspace problem. While hypercyclicity has been studied in-depth since the 1960s, the idea of subspace hypercyclic operators has only recently been introduced. An operator T is said to be subspace hypercyclic on the subspace M if there exists a vector x such that $\mathrm{Orb}(T, x) \bigcap M$ is dense in M. Given an arbitrary subspace M, there may exist operators that are hypercyclic or not, subspace hypercyclic on M or not, or any combination of these. We will explore the existence of invertible operators in each category, beginning with examples of operators on specific subspaces in ℓ^2 and then generalizing to an arbitrary subspace using isomorphism. [FC26104259]

[Joint with Rob Purcell and Citlalli Zamora]

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POSITIVE EXTENSIONS OF MATRICES INDEXED BY A HOMOGENEOUS TREE

Jeremy L. Garcia (garc4553@bears.unco.edu)
Laura M. Garcia (garciala@Grinnell.edu)
Kansas State University [Mentor:Dan Volok]

Abstract of Poster Presentation: Let T be a homogeneous tree of order q – that is, an acyclic, undirected, connected graph such that every node belongs to exactly $q + 1$ edges. Let T_n be a maximal subgraph of T with the property that the distance between any two nodes of T_n does not exceed n, and let $A = [a_{t,s}]_{t,s\in T_n}$ be a square positive definite matrix indexed by the nodes of T_n. The matrix A is said to be *isotropic* if $a_{t,s}$ depends only on the distance between the nodes t and s (in the case $q = 1$ this means that A is a real symmetric Toeplitz matrix). The positive extension problem for the matrix A consists in finding all such isotropic positive definite matrices B indexed by the nodes of $T_{n+1} \supset T_n$ such that the diagonal block of B corresponding to T_n coincides with A.

In this presentation we shall discuss a recursive solution of the positive extension problem formulated above. [GJ02142024]

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THE K-INDEPENDENCE POLYNOMIAL OF A GRAPH

Nicole A. Gin (ni156534@arbor.edu)
University of North Carolina at Asheville [Mentor:Patrick Bahls]

Abstract of Poster Presentation: A subset, S, of the vertices of a graph is called a *k-independent set* if the length of the shortest path between each pair of distinct vertices in S is at least $k+1$. In this talk, we generalize the familiar concept of the independence polynomial to that of the *k-independence polynomial*, which enumerates the k-independent sets in a given graph. We introduce a method of computing the k-independence polynomial of a graph G by using powers of G. We then consider when the k-independence polynomial of a graph is logarithmically concave. **[GN10181134]**

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PERFECT STATE TRANSFER, GRAPH PRODUCTS, AND EQUITABLE PARTITIONS

Benjamin C. Greenberg (greenber2@grinnell.edu)
Clarkson University [Mentor:Christino Tamon]

Abstract of Report Talk: We describe new constructions of graphs which exhibit perfect state transfer. Our constructions are based on generalizations of double cones and layered graphs. We also provide constructions of perfect state transfer graphs using the weak and lexicographic graph products. Some of our results include:

- Any double cone $\overline{K}_2 + G$, for an arbitrary connected graph G, has perfect state transfer if the weights on the cone edges are proportional to the Perron eigenvector of G.

- For an infinite family of n-vertex and k-regular graphs G, there is a circulant connection so the graph $K_1 + G_1 \circ G_2 + K_1$ has perfect state transfer, where $G_1, G_2 \in G$. In contrast, no perfect state transfer exists if a complete bipartite connection is used (even in the presence of weights).

- If G is either a $2n$-fold Cartesian product of K_2 or an n-fold Cartesian product of P_3, for any integer n, the weak product $G \times H$ has perfect state transfer if H is a regular graph with odd eigenvalues. This extends considerably the class of perfect state transfer graphs with arbitrarily large diameter.

The proofs we employ exploit elementary spectral properties of the underlying graphs. **[GB27172637]**

[Joint with Yang Ge, Oscar Perez]

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3D POLYTOPES OF MINKOWSKI LENGTH 1

Matthew Grimm (mgrimm2@kent.edu)
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Kent State University [Mentor:Jenya Soprunova]

Abstract of Report Talk: The Minkowski sum of two polytopes is the set of all pairwise sums of their points. In this project, we studied the Minkowski length $L(P)$ of a lattice polytope P, which is defined to be the largest number of non-trivial primitive segments whose Minkowski sum lies in P. The Minkowski length represents the largest possible number of factors in a factorization of polynomials with exponent vectors in P, and shows up in lower bounds for the minimum distance of toric codes.

Let $Q \subset P$ be the Minkowski sum of $L = L(P)$ lattice polytopes Q_i, each of Minkowski length 1: $Q = Q_1 + \cdots + Q_L$. In order to come up with lower bounds on the minimum distance of toric codes, one needs to bound I, the total number of interior lattice points of the polytopes $Q_1, \ldots, Q_L$. We prove that in the 3D case $I \leq 8$, which extends a previously known bound $I \leq 1$ for lattice polygons. Our methods differ substantially from those used in the two-dimensional case and we employ Kasprzyks classification of 3D Fano tetrahedra to prove our result. **[GM31121735]**

[Joint with Olivia Beckwith]

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PRINCIPAL G-BUNDLES, STACKS AND CLASSIFYING SPACES

Meng Guo (mengguo1@illinois.edu)
University of Illinois at Urbana-Champaign [Mentor:Matthew Ando]

Abstract of Report Talk: In the classical theory of fiber bundles, principal G-bundles over space X are classified up to isomorphism by homotopy class of maps from X into a classifying space BG. Much of the difficulty in this theory arises from the construction of BG as a colimit: so it is not easy to calculate $map(X, BG)$.

We describe an enlargement of the category of spaces to the 2-category of topological stacks. If X and Y are spaces, we may consider them as stacks, and then $map_{stacks}(X, Y) \cong map_{spaces}(X, Y)$. But the category of stacks includes additional objects: for example, the stack $\overset{G}{\underset{*}{\downdownarrows}}$. We show that $map_{stacks}(X, \overset{G}{\underset{*}{\downdownarrows}}) \cong$ principal G-bundles over$X$.(Here is equivalent of categories). Moreover, if X is a sufficiently nice space, then $\pi_0\ map_{stacks}(X, \overset{G}{\underset{*}{\downdownarrows}}) \cong \pi_0\ map_{spaces}(X, BG)$, where π_0 is homotopy classes. **[GM02135424]**

[Joint with Meng Guo]

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THE HOMOTOPY CRITICAL VALUES AND PARAMETERIZATION OF S^1 WITH DISCRETE CHAINS.

Alex T. Happ (ahapp@utk.edu)
Chris V. Carlile (ccarlil1@utk.edu)
University of Tennessee at Knoxville [Mentor:Conrad Plaut]

Abstract of Report Talk: Sormani and Wei in a series of papers investigated ε-covering spaces of geodesic spaces, and in particular Gromov-Hausdorff limits of compact Riemannian manifolds of non-negative Ricci curvature. The construction used by Sormani-Wei is based on a classical construction due to Spanier for locally path connected spaces. At about the same time, Berestovskii and Plaut developed a covering space construction for arbitrary metric spaces that uses discrete chains and homotopies, which was shown by graduate student Jay Wilkins to include the Sormani-Wei covers as a special case. Studying more general metric spaces is important even when considering pointed Gromov-Hausdorff convergence of non-compact geodesic spaces since the metric balls that are involved may not themselves be geodesic. Discrete chains and homotopies are also more amenable to use with Gromov-Hausdorff convergence than paths and traditional homotopies.

We developed direct methods to prove that for a circle C of circumference 1, the ε-cover in the sense of Berestovskii-Plaut ($\frac{3\varepsilon}{2}$-cover in the sense of Sormani-Wei) is trivial for $\varepsilon > \frac{1}{3}$ and the usual parameterization $\pi : \mathbb{R} \to C$ for $\varepsilon \leq \frac{1}{3}$. That is, $\frac{1}{3}$ is the only "homotopy critical value" for C. While this fact may be proved using traditional methods from covering space theory, we used only direct methods that do not require the existence of a simply connected covering space. Such methods may prove useful since many geodesic spaces do not have simply connected covering spaces. Moreover, Sormani-Wei left open the question of whether Gromov-Hausdorff limits of compact manifolds with non-negative Ricci curvature have simply connected covers. [HA02131422]

[Joint with Fred Byrd] Received: August 7, 2010

A SOLUTION TO THE LONELY RUNNER CONJECTURE FOR ALMOST ALL POINTS, AND A REFORMULATION

Christopher Horvat (chh48@pitt.edu)
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University of Pittsburgh [Mentor:Frank Beatrous]

Abstract of Report Talk: The Lonely Runner conjecture, a long-standing problem in Diophantine approximation, has been proven incrementally since it was introduced in 1967 by J.M. Wills. Recently, a short proof has been given for 8 total runners by Barajas and Serra. Sadly, each successive proof has not been shown to be extendable to higher n. We describe a proof of the conjecture for almost all velocities independent of the number of runners using dynamical systems theory, and describe various schemes which cover sections of the missing points. We further present a new problem which combines projective geometry and prime number theory, whose solution in the affirmative would prove the conjecture in its entirety. [HC31233702]

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TOTALLY GEODESIC SURFACES IN CHAIN LINKS

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Brigham Young University [Mentor:Jessica Purcell]

Abstract of Poster Presentation: The complement of a chain link with $n \geq 5$ link components is a hyperbolic 3-manifold. It is known that many interesting 3-manifolds are obtained by Dehn filling these chain link complements. However, for general n, explicit geometric information on these link complements is generally unknown. We derive this information for many values of n, including large values. Applications include bounds on slope length and Dehn fillings, and a proof that for large n these are not minimal volume. **[KJ02133312]**

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INSCRIBABILITY OF BIPARTITE POLYHEDRA

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Bard College [Mentor:Lauren Rose]

Abstract of Report Talk: We classify bipartite polyhedral graphs that are not inscribable in a sphere. Our main techniques are a relationship with Delauney tessellations and the methods of Dillencourt. In particular, we use Dillencourt's perfect matching criterion, which says that an inscribable bipartite polyhedron has a perfect matching containing any two disjoint edges. We find all bipartite polyhedra that fail this criterion. In fact, our classification completely characterizes all non-inscribable bipartite polyhedra with up to 18 vertices. However, we recently found two 20-vertex bipartite polyhedra that fail to be inscribable for reasons unrelated to the perfect matching criterion. We hope that this example will shed new light on the ongoing area of research for sufficient conditions for inscribability. **[KR01210217]**

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DISTRIBUTIONS OF EIGENVALUES OF REAL SYMMETRIC M-CIRCULANT MATRICES

Gene S. Kopp (gkopp@uchicago.edu)
Williams College [Mentor:Steven Miller]

Abstract of Report Talk: Random matrix ensembles model many phenomena, from nuclear energy levels to L-function zeros. The idea is to generate $N \times N$ matrices from some nice distribution and look at their spectra. As $N \to \infty$, the behavior of the eigenvalues of a "typical" matrix is close to the ensemble average; however, few ensembles are well-understood, and current theorems rarely illustrate transitions between ensembles. We study real symmetric m-circulant matrices with entries i.i.d.r.v. An m-circulant matrix has toriodal diagonals periodic of period m. We view m as a "dial" we can "turn" from the highly structured real symmetric circulant matrices, whose limiting eigenvalue density is Gaussian, to the ensemble of all real symmetric matrices, whose limiting eigenvalue density is a semicircle. The limiting eigenvalue densities p_m show a visually stunning convergence to the semicircle as $m \to \infty$. We prove this convergence.

In stark contrast to most studies of patterned matrices, our work provides explicit closed form expressions for the densities. We prove that p_m is the product of a Gaussian and a certain even polynomial of degree $2m-2$. The proof is by derivation of the moments from the eigenvalue trace formula. The key step is converting the central combinatorial problem in the calculation to an equivalent problem about Euler characteristic and algebraic topology. We then obtain the explicit formula using topology, generating functions, complex analysis, and Fourier analysis.

[KG29173809]

[Joint with Murat Kologlu]

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JULIA SETS CONVERGING TO THE FILLED BASILICA

Robert T. Kozma (rkozma@bu.edu)
Boston University [Mentor:Robert Devaney]

Abstract of Report Talk: In complex dynamical systems, previous results by Devaney et al. have shown that, for the family of singularly perturbed complex quadratic polynomial maps $z^2 + \lambda/z^2$, the Julia set converges to the closed unit disk as $\lambda \to 0$, i.e., to the filled Julia set of the map z^2. In this paper, we investigate the family of maps

$$F_\lambda(z) = z^2 - 1 + \frac{\lambda}{z^2},$$

where -1 is the center of the period 2 bulb of the Mandelbrot set. Using symbolic dynamics and Cantor necklaces, we prove that, as $\lambda \to 0$ along the positive real axis, the Julia set of F_λ converges to the filled basilica, i.e., to the filled Julia set of $z^2 - 1$. We conjecture similar behavior for an entire class of perturbed quadratic maps when we replace parameter -1 in F_λ by the center of any hyperbolic component of the Mandelbrot set. [KR22132548]

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BENFORD'S LAW FOR COEFFICIENTS OF MODULAR FORMS AND PARTITION FUNCTIONS

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University of Wisconsin-Madison [Mentor:Ken Ono]

Abstract of Report Talk: It has long been observed that many naturally occurring statistics and arithmetic functions have surprising properties. For example, if we examine the first digits of a sequence in base 10, instead of the a priori estimate that each digit should appear equally often we find that the first digit is a 1 about 6 times as often as it is a 9. Although this is a well-known heuristic, it has only been proven for a relatively small class of arithmetic functions. Using recent results of Ken Ono and Kathrin Bringmann on coefficients of harmonic Maass forms as well as classical theory of uniform distribution, we prove that the coefficients of an infinite class of modular forms satisfy the Benford distribution. This allows us to generate large classes of sequences which were previously unknown to be Benford. In particular, we will show this for the partition function $p(n)$ as well as numerous classes of natural partition functions.

[LR10012816]

[Joint with Theresa Anderson, Ruth Stoehr] Received: August 7, 2010

ON AN EXTREMAL PROBLEM OF POLYA

Tuan Le (kid10462000@yahoo.com)
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Abstract of Report Talk: The notion of transfinite diameter of planar sets was introduced by M. Fekete around 1920's. It plays an important role in classical complex analysis and is related to other well-known concepts such as the logarithmic capacity and Chebyshev polynomials. The transfinite diameter of a compact set in the complex plane is the limit of n-diameters of the set. For each $n \geq 3$, the n-diameter $d_n(E)$ of E is given by

$$d_n(E) = max\left\{ \prod_{1\leq i<j\leq n} |z_i - z_j|^{\frac{2}{n(n-1)}} \right\},$$

where the maximum is taken over all n-tuples $\{z_1, z_2, \cdots, z_n\}$ of points in E.

The following is the extremal problem of G. Polya: among all n-tuples $E = \{z_1, z_2, \cdots, z_n\}$ with $|z_i| \leq 1$, find one with the largest n-diameter. The solution of this problem, attributed to Polya, is given below.

$$d_n(E) \leq n^{\frac{1}{n-1}},$$

and the equality holds for n-tuples of equally spaced points on the boundary of the unit disc D. While investigating the transfinite diameter of sets of constant width, Prof. Zair Ibragimov was led to the following stronger version of Polya's problem: among all n-tuples $E = \{z_1, z_2, \cdots, z_n\}$ with $|z_i - z_j| \leq 2$ $(1 \leq i < j \leq n)$, find one with the largest n-diameter. He conjectured that the extremal configuration will also be the vertices of a regular n-gon, at least when n is odd. In this talk, we will show that this is indeed true in the cases of 5-tuples and 7-tuples, in which the vertices of the regular 5-gon and 7-gon both have the maximum 5-diameter and 7-diameter, respectively. Finally, we will also show that among all special 4-gon configurations with diameter 2 (e.g: square, rectangle, diamond, trapezoid and isosceles trapezoid), the trapezoid has the maximum 4-diameter.

[LT02023803]

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MEAN VALUE PROPERTIES OF HARMONIC FUNCTIONS ON THE SIERPINSKI GASKET

Anna Levina (alevina@kent.edu)
Cornell University [Mentor:Robert Strichartz]

Abstract of Poster Presentation: Fractals are a popular mathematical object these days, and over the past few years there has been much theory developed in the realm of Analysis that circles around these geometrically intriguing maps. But, in the strange world of fractal dimension, one should never look with Euclidean eyes. In this talk, I will go beyond the construction of the Sierpinski gasket and into some of this incredible, and often counter-intuitive, theory. We will begin with a brief introduction to Calculus on the Sierpinski Gasket, which I will then utilize in the presentation of my work on deriving a mean value property for this strange and beautiful shape. I will present a non-constructive proof of a theorem analogous to the mean value property of harmonic functions on a ball and discuss other current and future work on the topic. **[LA03101701]**

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HOMOTOPY CRITICAL VALUES OF METRIC SOLENOIDS

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University of Tennessee [Mentor:Conrad Plaut]

Abstract of Report Talk: Sormani and Wei studied the "covering spectrum" of a geodesic space using a classical construction of Spanier. The covering spectrum consists of those numbers ε for which the topological type of Spanier covers changes when considering open covers by balls of radius δ above or below ε. Berestovskii and Plaut developed a method of defining ε-covers for metric spaces that, unlike Spanier's construction, does not require local path connectedness. Their method involves discrete chains and homotopies rather than paths and traditional homotopies. An ε-chain in a metric space is defined to be a finite sequence of points $\{x_0, ..., x_n\}$ such that for all i, $d(x_i, x_{i-1}) < \varepsilon$. An ε-homotopy is a sequence of ε -chains such that each chain differs from its predecessor by either adding or removing a single point, while never changing the endpoints. The ε-covering space X_ε of X consists of all ε-homotopy classes of ε-chains starting at a fixed basepoint, with a suitable metric, and the "endpoint map" $\phi_\varepsilon : X_\varepsilon \to X$ is the ε-covering map of X (which is surjective when X is connected). When X happens to be a geodesic space, the ε -covers are the same as those studied by Sormani-Wei (replacing ε by $\frac{3\varepsilon}{2}$). However, the Berestovskii-Plaut construction allows one to consider spaces that are not locally path connected.

Among the most famous examples of compacta that are not locally path connected are the solenoids. These spaces can be given various natural metrics by considering them as the closures of paths in the infinite product T^∞ of circles with a suitable product metric. As in the case of geodesic spaces, one obtains a "spectrum" of covering spaces and corresponding "critical values" ε where the structure of the covering map changes. We have obtained results concerning computing these critical values and understanding the isometry type of the corresponding covering spaces. Our methods involve projecting onto "finite solenoids" that are circles with non-geodesic metrics. Most of the critical values of finite solenoids are represented by "gaps" corresponding to local minima of the distance function. We demonstrate that these critical values may, in turn, be used to approximate critical values of the corresponding (infinite) solenoid.
[LZ02151221]

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PERMISSIBLE PLANE EMBEDDINGS OF RIBBON GRAPH BLOW-UPS

Sarah J. Loeb (sloeb@hmc.edu)
Louisiana State University [Mentor:Neal Stoltzfus]

Abstract of Report Talk: Ribbon graphs have been constructed from link projections and their invariants used to construct link invariants. We study the question of reversing the construction. From each ribbon graph we construct a three-valent partially oriented graph (called the blow-up) by replacing each vertex by an oriented circle and attaching the edges around the circle according to the rotation system. A characterization of which blow-ups have permissible plane embeddings, i.e. those with embeddings where the orientation of the circles is determined by their nesting level, is given in terms of two forbidden topological minors: $K_{3,3}$ and a second partially oriented planar graph that does not have a permissible embedding. The proof uses Kuratowski's Theorem as well as an argument by cases based on connectivity. As an application we obtain a complete characterization of which ribbon graphs arise as a state smoothing of a link diagram. **[LS02140815]**

[Joint with Moshe Cohen] Received: August 9, 2010

THE RELATIVE TUTTE AND BOLLOBAS-RIORDAN POLYNOMIALS

Robert M. McCann (mccann.matt@gmail.com)
The Ohio State University [Mentor:Sergei Chmutov]

Abstract of Report Talk: The celebrated Thistlethwaite theorem expresses the Jones polynomial of a link as a specialization of the Tutte polynomial. This specialization is of the graph obtained from a checkerboard coloring of regions from the diagram of the link. There exists two independent techniques for generalizing this theorem to virtual links. The first approach uses the Bollobas-Riordan polynomial of a ribbon graph instead of the Tutte polynomial. The second approach uses a version of the relative Tutte ploynomial of a planar graph with a subset of distinguished edges, 0-edges. We will present a direct relation between these two approaches. Specifically, for each ribbon graph we consider its projection onto a plane and construct the previously mentioned planar graph with 0-edges. We will prove that the Bollobas-Riordan polynomial of the original ribbon graph is related to the relative Tutte polynomial of the planar graph obtained by this construction. This theorem will explain why both polynomials give the same Jones polynomial of a virtual link. **[MR31232521]**

[Joint with Brittany Albrinck, Drew Meyer, Timothy NeCamp, Steven Gubkin, Clark Butler] Received: August 9, 2010

ROOK POLYNOMIALS IN HIGHER DIMENSIONS

Rachel E. Moger-Reischer (remr001@bucknell.edu)
Grand Valley State University [Mentor:Feryal Alayont]

Abstract of Poster Presentation: A rook polynomial counts the placements of non-attacking rooks on a board that may have restricted positions. In two dimensions, a board is a collection of cells arranged in rows and columns. Equivalently, a board is a collection of pairs of integers corresponding to the cells. A rook placed on a cell can attack any other cell in the same row or column. These polynomials can be used to represent a variety of combinatorial problems including permutations with restrictions and matching problems. We will discuss rook polynomials on boards in dimensions higher than two by letting rook boards consist of d-tuples, where d is the dimension, and by letting rooks attack along hyperplanes. We will look at generalizations of families of boards whose rook numbers are directly calculable and correspond to famous number sequences to find new combinatorial interpretations of known sequences. In this talk, we will focus on results relating to the central factorial numbers, the Genocchi numbers, and Latin rectangles as well as boards that have been termed the staircase board and the diagonal board. This work was completed at the 2010 REU program at Grand Valley State University.

[MR01125853]

[Joint with Ruth Swift]

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THE DISTRIBUTION OF THE FRACTIONAL PARTS OF N/K, FOR RESTRICTED RANGES OF K

David W. Montague (davmont@umich.edu)
University of Michigan [Mentor:Jeffrey Lagarias]

Abstract of Report Talk: In the 1970's, Saffari and Vaughan published a series of papers about sets of the form $S_{N,f} := \{\{\{N/j\}\} \mid j \leq f(N)\}$, where $\{\{x\}\}$ represents the fractional part of x. Motivated by the occurence of sums of the form $\sum_{j=1}^{f(N)} \{\{N/j\}\}$ in a wide range of number theory problems, we worked on and extended their results. For example, for $f(N) = N^{\alpha}$, Saffari and Vaughn proved equidistribution for $1 > \alpha > 1/3$, but conjectured it held for all $1 > \alpha > 0$. Through an iterated form of van der Corput's method, we were able to prove this conjecture, and in this talk, we will explain the reasoning behind our approach and present a number of related distribution results that we obtained. **[MD02000459]**

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VISUALIZATION OF THE FAMILY OF EXPANDER GRAPHS, $SL_2(Q)$ WITH VARIOUS GENERATING SETS

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UC Santa Barbara [Mentor:Darren Long]

Abstract of Poster Presentation: A family of expanders is a family of networks with a bound on the size of the connections, which is increasing in size and difficult to disconnect. These graphs are useful for real-world networks that require high connectivity but have limited resources.

A construction of a particular family of expanders exists using 2-by-2 matrices over finite fields with determinant one and cleverly chosen sets of generators. There are known non-constructive proofs that both the diameter and the girth grow with the logarithm of the size of the graph.

We ultimately wish to find better visualizations of expander graphs other than through algebraic constructions, bounds on various properties of the graphs, and planar representations. In order to gain an intuitive understanding of general families of expanders, we examined the Cayley graph of $SL_2(q)$ with various generators, and concrete ways to visualize them. We explore different ways to partition the graph, in order to define the graph as more concrete geometric objects. **[OS02155850]**

[Joint with Lauren Grimley, Nathan Saritzky, Eric Demer]

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PACKETS, SOLVING SYMMETRIES AND SUDOKU

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Harrison C. Chapman (hchapman@bowdoin.edu)
James Madison University [Mentor:Elizabeth Arnold]

Abstract of Report Talk: In a typical Sudoku puzzle, a number of initial clues are given, and the solver uses strategies to fill in the remaining clues to complete the board. In this talk, we shift the focus of study from clues to what we call packets. A packet gives information about what clues cannot be in a cell. Using a brute force computer search with appropriate reductions, we answer the question, “what is the minimum number of packets needed to describe a puzzle with a unique completion?” Packets are also intimately related to the Boolean system of polynomial equations used to describe the constraints of a Sudoku puzzle. We show how they can be used to more efficiently calculate a Gröbner basis of the ideal generated by this system of equations. Packets can be used to algebraically construct solving symmetries which mimic the human strategies involved in solving Sudoku puzzles. Solving symmetries are functions which manipulate a puzzle while maintaining the same solutions. We prove that these solving symmetries form a group which act on the set of Sudoku puzzles. We explore the structure of this group and discuss a long-standing open problem in Sudoku. **[RM28145143]**

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LOG-LINEAR ODES AND APPLICATIONS TO THE RICCI FLOW FOR HOMOGENEOUS SPACES

Mary E. Russell (russellm@canisius.edu)
University of Arizona [Mentor:Tracy Payne]

Abstract of Poster Presentation: The Ricci flow is a geometric evolution that tries to evenly distribute the Ricci curvature throughout the manifold. On a homogeneous space, one can consider the bracket flow, an analog of the Ricci flow that evolves the structure constants of the Lie algebra. We analyzed a class of ODE arising from the bracket flow on nilpotent and solvable Lie algebras. For two dimensional systems, we completely classified the possible geometric behavior. In all dimensions, we found sufficient conditions for collapsing, and necessary and sufficient conditions for the existence of soliton metrics. **[RM01193548]**

[Joint with Michael Kreisel]

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HOM COMPLEXES AND THE LEFSHETZ FIXED POINT THEOREM

Mychael Sanchez (cmendnba@nmsu.edu)
New Mexico State University [Mentor:Daniel Ramras]

Abstract of Poster Presentation: Given graphs G and H, The Hom complex $\mathrm{Hom}\,(G,H)$ is a simplicial complex formed by the graph homomorphisms from G into H. The Hom complex allows one to prove results about graph colorings and chromatic number. We are investigating the topology on the Hom complex by applying the Lefshetz Fixed Point Theorem. By studying automorphisms of G and H, we obtain conditions under which $\mathrm{Hom}\,(G,H)$ is not contractible.
[SM30162728]

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PROVING THAT A GROUP OF HYPERBOLIC ISOMETRIES IS DISCRETE

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Columbia University [Mentor:Maksim Lipyanskiy]

Abstract of Report Talk: Given a discrete group of hyperbolic isometries, we can quotient 3-space by the group action and form a hyperbolic 3-manifold with fundamental group G. Conversely, how can we decide whether group of hyperbolic isometries is discrete? In this talk, we present a new computer-assisted method for proving a subgroup of hyperbolic isometries is discrete using Poincaré's theorem for fundamental polyhedra. We first construct a Dirichlet domain D for G using our computer program and use this to approximately verify the hypotheses of Poincaré's Theorem to high accuracy. In order to *exactly* verify the hypotheses, it turns out that when the Dirichlet polyhedron has certain nice geometric properties we can utilize the group structure of G to rigorously establish our result. Amazingly, we can show that these properties hold for almost every choice of center of D in hyperbolic 3-space. We will begin by introducing the geometry of discrete groups, Dirichlet domains, and Poincaré's Theorem and we will close the talk by showing examples of how our method is used to prove certain knot complements are hyperbolic. **[SK02154156]**

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CLIFFORD ALGEBRA FILTRATIONS AND SUPERSYMMETRY

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Bard College [Mentor:Gregory Landweber]

Abstract of Report Talk: Our presentation provides a background on supersymmetric concepts in mathematics and physics. The research pursued is focused on objects called Clifford algebras, a generalization of the transitions from the real to complex numbers and from the complex numbers to the quaternions. We study Clifford algebras using Adinkras, continuing the work carried out by Doran, Faux, Gates, Hubsch, Iga, and Landweber. This paper targets $\mathbb{Z}_2$-graded filtrations of irreducible representations of the Clifford algebra, defining equivalence maps between filtrations and investigating the resulting equivalence relations. The equivalence classes for $N = 0, 1, 2, 3$, and 4 are fully classified, and significant progress is made toward the classification of $N = 7$ and 8. For $N = 0, 1, 2, 3, 4$ and 8, the relevant equivalence maps are constructed explicitly. The paper culminates in a general condition: all equivalences may be constructed from left multiplication and conjugation by an element of $Spin(N)$ (up to a constant). $Spin(N)$ is a double cover of the group $SO(n)$ of orthogonal transformations with determinant one. **[SS02004227]**

[Joint with Zhexiu Tu]

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METRIC AND TOPOLOGICAL SPACES FOR STUDYING PARTITION REGULARITY

Liam Solus (lsolus@oberlin.edu)
Mt. Holyoke College [Mentor:Jillian McLeod]

Abstract of Report Talk: In 1939, Rado showed that any complex matrix is partition regular over $\mathbb{C}$ if and only if it satisfies the columns condition. Recently, Hogben and McLeod explored the linear algebraic properties of matrices satisfying partition regularity. Here, we further the discourse by generalizing the notion of partition regularity beyond systems of linear equations to topological surfaces and graphs. We begin by defining, for an arbitrary matrix Φ, the metric space $(M_\Phi,\ \delta)$, where M_Φ is the set of all matrices equivalent to Φ that are (not) partition regular if Φ is (not) partition regular; and for elementary matrices, E_i and F_j, we let $\delta(A, B) = min\{m = l + k | B = E_1 \ldots E_l A F_1 \ldots F_k\}$. Subsequently, we illustrate that partition regularity is in fact a local property in the topological sense, and uncover some of the properties of partition regularity from this perspective. We then use these properties to establish that all compact topological surfaces are partition regular, but not all graphs. **[SL13151720]**

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ON THE DISTRIBUTION OF ORDERS OF BASIS FOR $\mathbb{Z}_N$

Wesley J. Souza (wsouza@ucsc.edu)
UC Santa Barbara [Mentor:Maribel Bueno]

Abstract of Report Talk: Let $S \subseteq \mathbb{Z}_n$. For $k \in \mathbb{N}$ we denote by kS the set

$$kS := \{a_1 + \cdots + a_k \mid a_i \in S\}.$$

S is said to be a basis for $\mathbb{Z}_n$ if there exists a $k \in \mathbb{Z}_n$ such that $kS = \mathbb{Z}_n$. The smallest such k is said to be the order of S denoted by $order(S)$. In our project we study the set $E_n = \{order(S) \mid S \subseteq \mathbb{Z}_n\} \subset \mathbb{Z}_n \setminus \{0\}$, whose elements are not well known. Determining the distribution of E_n in $\mathbb{Z}_n$ has been an open problem since 1974.

We call the j^{th} box, B_j, of $\mathbb{Z}_n$ the set of integers in the interval

$$\left[\left\lfloor \frac{n}{j} \right\rfloor - 1, \left\lfloor \frac{n}{j} \right\rfloor + j - 2\right]$$

and we conjecture that if $S \subseteq \mathbb{Z}_n$ is a basis, then $order(S) \in [1, \lceil\sqrt{n}\rceil] \cup \bigcup_{j=1}^{\lceil\sqrt{n}\rceil} B_j$. We have proven that all integers in $[1, \lceil\sqrt{n}\rceil]$ are attained by bases of $\mathbb{Z}_n$. In addition, we determined the distribution of the orders in B_j for $j \in \{1, 2, 3\}$. Some progress has been achieved regarding the distribution in the fourth box.

This problem has equivalent statements in terms of Boolean circulant primitive matrices and circulant digraphs. Applications of this question appear in diverse areas such as Markov processes, coding theory, and quantum information. **[SW02141643]**

[Joint with Kuan-Ying Fang, Raquel Barata, Samantha Fuller]

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FAMILIES OF HYPERPLANES IN THE REAL AND COMPLEX HYPERBOLIC PLANE

Melanie I. Stam (mstam@gatech.edu)
Georgia Institute of Technology [Mentor:Igor Belegradek]

Abstract of Poster Presentation: We use the hyperboloid model of the real and complex hyperbolic plane and generate families of lines by considering $\{\bar{x} \mid \|\bar{x}\| = 1, x_i \in L\ \forall i\}$, where L is a certain lattice in $\mathbb{R}^{1,2}$ or $\mathbb{C}^{1,2}$. Considering the intersection of the hyperboloid with the orthogonal complements of these $\bar{x}$ yields infinite families of geodesics. We then imagine these families as a graph, where each geodesic is a vertex and two vertices are connected by an edge if the geodesics they represent intersect or are asymptotic. In the real case, we consider the lattice of integers and decide that it is connected only if we consider asymptotic geodesics to be connected. In the complex case we consider the lattice of Gauss and Eisenstein integers, deciding that both are connected only if we include connections between asymptotic vertices. The study of such families of lines are interesting not only because they are nice groups of lines which arise quite naturally from the hyperboloid model, but also because their properties can be helpful in the study of the quotient group of the hyperbolic plane by the symmetry group of the family of lines. **[SM01232426]**

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A UNIVERSAL CAUCHY-RIEMANN FUNCTION

Louis-Philippe Thibault (lp.thibault@gmail.com)
Universite de Montreal [Mentor:Paul Gauthier]

Abstract of Report Talk: An entire function f is said to be universal if the set of its translates $\{f(z+c) : c \in \mathbb{C}\}$ is dense in the space of all entire functions on $\mathbb{C}$. In 1929, G.D. Birkhoff showed the existence of a universal entire function on the complex plane. This theorem is very surprising and was later generalized in many ways. For example, the existence of a universal holomorphic function in $\mathbb{C}^n$ has recently been shown, but there is still open questions about possible other generalizations. In this talk, we prove a new interesting analogue of Birkhoff's theorem for Cauchy-Riemann functions on the mixed euclidean space $\mathbb{C}^n \times \mathbb{R}^m$, that is, continuous complex-valued functions $g(z, \eta)$ on $\mathbb{C}^m \times \mathbb{R}^n$, which are holomorphic in $\mathbb{C}^n$ for each fixed $\eta \in \mathbb{R}^m$. Namely, there exists a Cauchy-Riemann function $f(z, \eta)$ on $\mathbb{C}^m \times \mathbb{R}^n$ having the remarkable property that its translates are dense in the space of all Cauchy-Riemann functions on $\mathbb{C}^m \times \mathbb{R}^n$. We will first need to prove a Weierstrass-type approximation theorem on compact subsets of $\mathbb{C}^m \times \mathbb{R}^n$.

[TL26210549]

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TROPICAL DETERMINANTS AND CHEATING WHEN SOLVING RUBIKS CUBE

Thomas J. Dinitz (tdinitz@students.colgate.edu)
Kent State University [Mentor:Jenya Soprunova]

Abstract of Report Talk: Consider the usual Rubik's cube with 9 square stickers on a side, each sticker colored in one of six colors. Instead of solving it in the normal way (by rotating faces), we want to solve it by peeling off and replacing the stickers. In this presentation we will address the natural question: What is the maximum number of stickers you would ever need to peel off and replace to solve the Rubik's cube? We first show that this problem can be translated to the language of matrices. Let A be a 6 by 6 matrix whose(i, j)th entry is the number of squares of color i in face j of the cube. The matrix A has all its row and column sums equal to 9 (as there are 9 stickers of each color, and each face of a cube has 9 stickers). The number of stickers that we do not need to peel off and replace is equal to the sum of entries in the largest transversal of A (called the tropical determinant of A). We would like to find such a matrix A with the lowest possible tropical determinant as this would provide an example of a Rubik's cube which needs the most number of stickers replaced. Stated more formally, our initial problem reduces to finding a sharp lower bound on the tropical determinant of 6 by 6 integer matrices A with non-negative entries and row and column sums equal to 9. We solve this problem as well as the general problem of n-by-n matrices with row and column sums equal to m.

[TD26105631]

[Joint with Matthew Hartman]

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ON SUBBARAO'S CONJECTURE ON THE PARITY OF THE PARTITION FUNCTION

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Rebecca Hoberg (rahoberg@gmail.com)
University of Wisconsin-Madison [Mentor:Ken Ono]

Abstract of Report Talk: Let $p(n)$ denote the ordinary partition function. Subbarao conjectured that in every arithmetic progression $r \pmod t$ there are infinitely many integers N (resp. M) $\equiv r \pmod t$ for which $p(N)$ is even (resp. odd). By the early '90s, Subbarao's Conjecture had only been verified for arithmetic progressions with modulus $t \in \{1, 2, 3, 4, 5, 6, 8, 10, 12, 16, 20, 40\}$. Until recently, the only infinite class of moduli t for which Subbarao's Conjecture was proven was $t = 2^s$. We proved Subbarao's conjecture for all moduli t of the form $m \cdot 2^s$ where $m \in \{1, 5, 7, 17\}$. To obtain this theorem we made use of recent results on the nilpotent action of Hecke algebras on certain spaces of modular forms modulo 2.

[TG11204757]

[Joint with Rebecca Hoberg, Mckenzie West]

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COPS AND ROBBERS ON PLANAR GRAPHS

Silviya D. Valeva (valev20s@mtholyoke.edu)
University of Minnesota IMA [Mentor:Andrew Beveridge]

Abstract of Report Talk: In the game of *Cops and Robbers* on a graph $G = (V, E)$, k cops try to catch a robber. On the cop turn, each cop may move to a neighboring vertex or remain in place. On the robber's turn, he moves similarly. The cops win if there is some time at which a cop is at the same vertex as the robber. Otherwise, the robber wins. The minimum number of cops required to catch the robber is called the *cop number* of G, and is denoted $c(G)$. The game of Cops and Robbers has applications in robotics and in search and rescue operations.

A classic result of Aigner and Fromme shows that if G is planar then $c(G) \leq 3$. We characterize the following families of planar graphs as having $c(G) \leq 2$: series parallel graphs, outerplanar graphs, maximal 2-outerplanar graphs, and maximal planar graphs with maximum degree at most 5. We also show that every graph G with $|V| \leq 9$ has $c(G) \leq 2$. This bound is tight, since the Petersen graph (on 10 vertices) requires 3 cops. **[VS28174048]**

[Joint with Aaron Maurer, John McCauley]

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DEDUCING A Q-ANALOG FOR FLECK'S CONGRUENCE

Robert M. Walker (rwalker6@uiuc.edu)
University of Illinois at Urbana-Champaign [Mentor:Andrew Schultz]

Abstract of Report Talk: The binomial coefficient $\binom{n}{m}$ participates in Fleck's congruence: Let p be a prime number, and let n and j be integers such that $0 \leq j \leq p-1 < n$. Then,

$$\sum_{m \equiv j \bmod p} \binom{n}{m} (-1)^m \equiv 0 \pmod{p^k},$$

where $k = \lfloor (n-1)/(p-1) \rfloor$ (here, $\lfloor x \rfloor$ denotes the largest integer less than or equal to x).

Meanwhile, a q-analog is a theorem or identity in the variable q that yields a familiar mathematical object or result in the corresponding limit, as q converges to 1. For example, the q-binomial coefficient $\binom{n}{m}_q$ is a polynomial generalization of the binomial coefficient $\binom{n}{m}$, which equals the latter when $q = 1$.

We ask if a q-analog of Fleck's congruence follows from swapping out the binomial coefficients in the relevant sum for their q-analog counterparts? By (a) constructing a function in Mathematica to compute q-ified versions of Fleck's sum, and (b) studying the factorizations of these sums, we isolated several short-term patterns in the appearance of certain factors, and formulated conjectures based on those patterns. In example, Conjecture 1 reads: Let $n, k \in \mathbb{Z}^+$ such that $n = 2k$ is even. Then,

$$\sum_{m \equiv 1 \bmod 2} \binom{n}{m}_q (-1)^m \equiv 0 \pmod{[2]_{q^k}},$$

where $[2]_{q^k} = 1 + q^k$ approaches 2, as q approaches 1. Appealing to number theoretic notions and results (both elementary and more advanced), we have provided theoretical proofs for six conjectures, including Conjecture 1. We have one outstanding conjecture left to prove. Finally, we can consider implications for further research, given our successes. **[WR02153600]**

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FROM FIBONACCI NUMBERS TO CENTRAL LIMIT TYPE THEOREMS

Yinghui Wang (yinghui@mit.edu)
Williams College [Mentor:Steven Miller]

Abstract of Report Talk: A beautiful theorem of Zeckendorf states that every integer can be written uniquely as a sum of non-consecutive Fibonacci numbers $\{F_n\}_{n=1}^{\infty}$. Lekkerkerker proved that the average number of summands for integers in $[F_n, F_{n+1})$ is $n/(\varphi^2+1)$, with φ the golden mean. We prove the following massive generalization: given nonnegative integers $c_1, c_2, \dots, c_L$ with $c_1, c_L > 0$ and recursive sequence $\{H_n\}_{n=1}^{\infty}$ with $H_{n+1} = c_1 H_n + c_2 H_{n-1} + \cdots + c_L H_{n+1-L}$ $(n \geq L)$, $H_1 = 1$ and $H_{n+1} = c_1 H_n + c_2 H_{n-1} + \cdots + c_n H_1 + 1$ $(1 \leq n < L)$, every positive integer can be written uniquely as $\sum a_i H_i$ under natural constraints on the a_i's, the mean and the variance of the numbers of summands for integers in $[H_n, H_{n+1})$ are of size n, and the distribution of the numbers of summands converges to a Gaussian as n goes to the infinity. Previous approaches were number theoretic, involving continued fractions, and were limited to results on existence and, in some cases, the mean. By recasting as a combinatorial problem and using generating functions and differentiating identities, we surmount the limitations inherent in the previous approaches.

Our method generalizes to a multitude of other problems. For example, every integer can be written uniquely as a sum of the $\pm F_n$'s, such that every two terms of the same (opposite) sign differ in index by at least 4 (3). We prove similar results as above; for instance, the distribution of the numbers of positive and negative summands converges to a bivariate normal with computable, negative correlation, namely $-(21-2\varphi)/(29+2\varphi)$. **[WY28164055]**

[Joint with Gene Kopp, Murat Kologlu]

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APPLICATIONS OF AND ALTERNATIVES TO ALGORITHM X FOR THE EXACT COVER PROBLEM

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Abstract of Poster Presentation: Currently, the best algorithm for solving the NP-Complete Exact Cover problem is Donald Knuth's Algorithm X. Motivated by the algorithm's efficiency, our work investigates conversion of other NP-Complete problems to Exact Cover form. We present an improvement on Sage's reformulation of Vertex Coloring problems to Exact Cover problems by using partial complete graph decompositions. This improvement generalizes the observation that the popular logic puzzle Sudoku is both an Exact Cover problem and a Vertex Coloring problem. After proving that chromatic polynomials cannot be used to find colorings of most arbitrarily partially-colored graphs, we turn to our method of conversion to find solutions. To solve the converted Vertex Coloring problem as well as the general Exact Cover problem, we present a detailed explanation of Algorithm X followed by an investigation into a new algorithm we developed to reduce any Exact Cover problem to a smaller exponential problem. Future research will address the comparative efficiency of these two algorithms in various conditions. **[WB29135336]**

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ON THE SPLITTING FIELDS OF GENERALIZED RIKUNA POLYNOMIALS

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Abstract of Report Talk: For each $\ell \geq 3$, Rikuna discovered a polynomial F_ℓ over the function field $K(T)$ whose Galois group is $\mathbb{Z}/\ell\mathbb{Z}$, for any field K satisfying some mild conditions. Komatsu recently generalized the classical Kummer theory to also encompass cyclic extensions arising from Rikuna polynomials.

Fix an $\ell \geq 3$. For each $m \geq 1$, we introduce the *m-th generalized Rikuna polynomial* r_m, which is formed from the m-th iteration of a rational function related to the ℓ-th Rikuna polynomial. Define K_m to be the splitting field of r_m over $K(T)$. By a result of Cullinan on the arithmetic dynamics of iterated rational functions, the tower of K_m's will be ramified at finitely many primes of $K(T)$.

In our work, we study the tower of K_m's for prime ℓ. We show that for all $m \geq 1$,

$$\mathrm{Gal}(K_m/K(T)) \simeq \mathbb{Z}/\ell^{m-v}\mathbb{Z} \rtimes \mathbb{Z}/\ell^m\mathbb{Z}$$

for $v = \min\{b, m\}$, where $b \in \mathbb{N} \cup \{\infty\}$ depends only on K. We also show that only one prime of $K(T)$ ramifies in the tower of K_m's, and explicitly find this prime. Using the Riemann-Hurwitz formula, we determine that K_m has genus 0 for all $m \geq 1$. This implies that K_m has class number 1 for all $m \geq 1$. We present conjectures on the extension of these results to general $\ell \geq 3$.

[WF02024443]

[Joint with Hannah Hausman, Sean Pegado]

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THE WEAK BRUHAT ORDER AND SEPARABLE PERMUTATIONS

Fan Wei (fanfanfanweiweiwei@gmail.com)
Massachusetts Institute of Technology [Mentor:Richard Stanley]

Abstract of Report Talk: Let $\mathfrak{S}_n$ denote the symmetric group of all permutations of $1, 2, \ldots, n$, partially ordered by the weak (Bruhat) order. Thus for any permutation $w \in \mathfrak{S}_n$, the rank $\ell(w)$ of w in $\mathfrak{S}_n$ is the number of inversions in w. It follows that the rank-generating function of $\mathfrak{S}_n$ is

$$F(\mathfrak{S}_n, q) = \sum_{w \in \mathfrak{S}_n} q^{\ell(w)} = \boldsymbol{(n)!},$$

where $\boldsymbol{(n)!} = \boldsymbol{(1)(2)\cdots(n)}$ and $\boldsymbol{(i)} = 1 + q + q^2 + \cdots + q^{i-1}$.

For any $w \in \mathfrak{S}_n$, we define two graded posets associated with w: $\Lambda_w = \{v \in \mathfrak{S}_n : v \leq w\}$ and $V_w = \{v \in \mathfrak{S}_n : v \geq w\}$. In V_w, we define the rank of $v \in V_w$ to be $\ell(v) - \ell(w)$.

In general, the rank-generating functions of Λ_w and V_w are messy. However, in this talk we will show that if w is a separable permutation, i.e., 3142-avoiding and 2413-avoiding, then there is the surprising formula

$$F(\Lambda_w, q)F(V_w, q) = \boldsymbol{(n)!}.$$

Moreover, we define a bijection $\varphi\colon \Lambda_w \times V_w \to \mathfrak{S}_n$ satisfying $\ell(u) + \ell(v) - \ell(w) = \ell(\varphi(u, v))$, and we give a convenient method to find an explicit formula for $F(\Lambda_w, q)$ and $F(V_w, q)$. In particular, we find that if w is a 231-avoiding, or 213-avoiding, or 312-avoiding, or 132-avoiding permutation, a more direct expression for $F(\Lambda_w, q)$ and $F(V_w, q)$ can be given. We also deduce from our formula for $F(\Lambda_w, q)$ and $F(V_w, q)$ that the posets Λ_w and V_w are rank-symmetric and rank-unimodal for all separable permutation w.

[WF27214547]

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A SOLVABLE HYPERBOLIC FREE BOUNDARY PROBLEM MODELING TUMOR RECURRENCE

Dian Yang (yangd1989@gmail.com)
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Abstract of Poster Presentation: Recently Tian, Friedman et. al. have developed a model about brain tumor glioblastoma recurrence after resection [TF]. The model is a hyperbolic free boundary problem with two nonlinear partial differential equations. It integrates clinical data and suggests some optimal protocols of radiotherapy and chemotherapy by numerical study. In this paper, we analyzed this hyperbolic system. We proved the local and global existence and uniqueness of the solution. It is well known that most free boundary problems are unsolvable in terms of explicit analytical solutions. However, our free boundary problem is solvable, and we found the explicit solution by using backward characteristic curve method. This explicit solution agrees with our numerical simulations. Another interesting property is that the system can be treated as an hyperbolic system defined in an infinite domain and the initial function with one first-type of discontinuity.

[TF] Jianjun Paul Tian, Avner Friedman, Jin Wang and E. Antonio Chiocca, *Modeling the effects of resection, radiation and chemotherapy in glioblastoma*, Journal of Neuro-Oncology, **91** No. 3 (2009), pages 287-293. [YD26205424]

[Joint with Wang Jin, Assistant Professor, Old Dominion University] Received: August 11, 2010

A DISCRETE CONSIDERATION OF ALEKSANDROV'S PROJECTION THEOREM

Hannah D. Yee (hyee@hillsdale.edu)
Kent State University [Mentor:Dmitry Ryabogin]

Abstract of Report Talk: Aleksandrov's Projection Theorem states that if the areas of projections of two centrally symmetric, convex bodies K and L are equal in every direction, then K and L are translates of one another. We will consider the nonexistence of a discrete analogue to Aleksandrov's Theorem given polytopes in $\mathbb{Z}^2$ where the numbers of projection points of two polytopes are equal in every direction. Counterexamples from previous research on this problem in discrete tomography will be discussed, as well as polytopes we constructed based on observed characteristics of the known counterexamples. A new proof of an analogue in $\mathbb{Z}^2$ to Minkowski's Uniqueness and Existence Theorems will also be given, as in continuous space these are key components to the proof of Aleksandrov's Projection Theorem. [YH30171040]

[Joint with Matthew Alexander and Sarah Mullin] Received: August 9, 2010

EXPLORING THE EFFECTS OF SURFACE TRANSFORMATIONS ON CLOSED CURVES

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Abstract of Report Talk: We study the effect of different surface transformations on generators of the fundamental group of the surface. Since every curve on the surface can,up to deformation, be described in terms of the generators, we know what happens when applying our transformation to any particular curve by studying the generators. One of our transformations is defined on the disk with three punctures. We keep switching around those punctures: left over the middle, right over the middle, then repeat. One interesting fact is that after applying our transformation, the number of the branches under each puncture can be described by the sequence of Fibonacci numbers. Consider a thin cylinder on a surface. Fix one end of the cylinder and twist the opposite end of the cylinder by 2π. Extending by the identity outside the thin cylinder defines a transformation of the surface called the Dehn Twist. Our second transformation is made by composing two different Dehn twists whose thin cylinders intersect essentially. We try to find patterns in the orbit $(A, f(A), f(f(A)), \ldots)$ of a non-self-intersecting curve A obtained by iterated applications of a surface transformation f. In particular, we study the combinatorial lengths along the orbit. (If we describe a curve up to deformation by a word in the generators of the fundamental group of the surface, the combinatorial length of a curve is the minimal number of letters required.) The common property of these two examples of transformations is that no power of either one has a fixed curve. This implies the stronger property that the surfaces are stretched in one set of directions and compressed in another set of directions by such a transformation. **[YR30174900]**

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MAPS AND INDICATRICES OF KNOTS

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Abstract of Summary Talk: A knot is a simple closed curve in space, and as such, we can apply the tools of differential geometry to it. From it, we can generate the tangent indicatrix, the normal indicatrix, and the binormal indicatrix on the unit sphere. The SMALL09 knot theory group defined the bridge map of a knot and proved the bridge map is the union of the binormal indicatrix, its antipodal curve, and some great circle arcs. We will use the shape of the binormal indicatrix to determine a bound for the superbridge index of a knot. It serves as an attempt to determine all the 3-superbridge knots. We will also extend this idea to generate the alpha-map for a knotted torus in space. We will see lots of pictures and no particular background is assumed. **[ZL02145555]**

[Joint with Colin Adams, Thomas Crawford, Kyler Siegel, Stephanie Jensen, Michelle Chu] Received: August 7, 2010

AN INVESTIGATION OF THE GEOMETRY OF TOPOLOGICAL SURFACES WITH COMBINATORICS AND MULTI-WORDS

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Abstract of Poster Presentation: The geometry of a topological surface is subtle and difficult to capture. To study the orientable surfaces the pair of pants and the punctured torus, we study the curves on these surfaces. If one curve on the surface can be continuously deformed into another, then they can be identified into the same free homotopy class. It is known that each free homotopy class can be uniquely represented by a cyclic reduced word. Each cyclic reduced word consists of letters A, a, B, b with the identities $Aa = aA = id$ and $Bb = bB = id$ (i.e. words do not contain Aa, aA, Bb, bB). Also, words are identified up to cyclic permutation, for instance, $Abb = bAb = bbA$. A multi-word is defined to be a collection of cyclic reduced words. The length of a multi-word is the sum of the number of letters of each cyclic reduced word in the collection. To investigate the topological properties of the surfaces, we study the minimal intersection number (up to continuous deformation) of each multi-word and look for relationship between the length L of a multiword and its minimal intersection number N. With this powerful combinatorial tool, unapparent but nontrivial properties of the geometry and topology of the surface can be explored. **[ZJ30202448]**

[Joint with Chao Xu (programming support)]

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APPROACHING CONSTRUCTABILITY OF TRIANGLES

Stefanie R. Zukowski (zukowssr@muohio.edu)
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Abstract of Report Talk: A natural problem in geometry is to find constructions, using only a ruler and a compass, of triangles with given positions of certain three points, such as the feet of the three altitudes. Problems of this type can be very easy, such as if given midpoints of the three sides, or very challenging, several remaining unsolved.

These problems have been studied throughout the history of geometry, as when Euler proved that a triangle with given orthocenter, incenter, and circumcenter, in general, is not constructible. In 1982 Wernick compiled a list of 139 problems of this type providing solutions to several of them, but leaving 41 problems unresolved. In 1995 Meyers published solutions to several more of Wernick's problems, still leaving 20 open problems. Since then several mathematicians contributed both new solutions and new problems.

This summer I proved that 8 previously unresolved problems are non-constructible. In addition to geometry, I used complex analysis and abstract algebra. I will present a general method of approach to these problems, which I developed in my research. **[ZS31214814]**

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