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Frechet Differentiability in Optimal Control of the Stefan Problem

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Abstract of Poster Presentation: We prove Fréchet differentiability and derive the formula for the Fréchet gradient in an optimal control of the Stefan type free boundary problem for the general second order linear parabolic PDE:

$$\frac{\partial}{\partial x}\Big(a(x,t)\frac{\partial u}{\partial x}\Big)+b(x,t)\frac{\partial u}{\partial x}+c(x,t)u-\frac{\partial u}{\partial t}=f(x,t)$$

where $u(x,t)$ is the temperature function. The reaction coefficient c, unknown free boundary, and boundary heat flux are components of the control vector, and cost functional consists of the L_2-declination of the trace of the temperature at the final moment, temperature at the fixed boundary and final position of the free boundary from available measurements. This problem arises in mathematical modeling and control of various phase transition processes in thermophysics, biomedical engineering, fluid mechanics. We follow a new variational formulation developed in *U. G. Abdulla, Inverse Problems and Imaging, 7,2(2013),307-340*.
In this project we employ the infinite dimensional calculus setting in Besov-Hilbert space framework. We introduce the notion of adjoint problem, as an infinite dimensional analogy of the classical Lagrange multupliers in finite dimensional constrained optimization problems. With the delicate use of sharp embedding theorems in fractional Sobolev-Besov spaces we derive the formula for the Fréchet gradient expressed in terms of the traces of the state vector and the solution of the adjoint problem. The result implies the necessary optimality condition in the form of variational inequality in Besov spaces. We use Fréchet differentiability result and necessary optimality condition for implementation of the projective gradient method in Hilbert spaces setting for the numerical solution of the problem.

[Joint work with Nadab Wubshet, Rajendra Beekie, Saleheh Seif, Vlad Bukshtynov, Jim Jones] Received: August 16, 2016