

# YOUNG MATHEMATICIANS CONFERENCE 2016

*The Ohio State University, August 19-21*

## SECOND MINIMAL ORBITS, SHARKOVSKI ORDERING, AND UNIVERSALITY IN CHAOS

**Rashad U Abdulla**

(abdullar@sas.upenn.edu)

**Emily Ribando-Gros and Muhammad Abdulla**

(eriban1@osu.edu)

*Florida Institute of Technology*

[Mentor: Ugur Abdulla]

**Abstract of Report Talk:** We prove the outstanding conjecture on the number of second minimal odd periodic orbits of the continuous endomorphisms on the real line. An  $n$  periodic orbit of the map is called a second minimal if  $n$  is the successor of the minimal orbit of the map in Sharkovski ordering. It is proved that for any integer  $k \geq 3$  there are  $4k - 3$  types of second minimal  $2k + 1$ -orbits with accuracy up to inverses. Our proof presents fine classification of the second minimal odd orbits in terms of cyclic permutations and directed graphs. The result is applied to the problem on the distribution of superstable periodic windows within the chaotic regime of the bifurcation diagram of the one-parameter family of unimodal maps. It is revealed that by fixing the maximum number of appearances of the periodic windows there is a universal pattern of distribution. In particular, the first appearance of all the orbits is always a minimal Stefan orbit, while the second appearance is always a second minimal orbit with precisely Type 1 digraph according to our classification. The reason for the relevance of the Type 1 minimal orbit is the fact that the topological structure of the unimodal map with single maximum is equivalent to the structure of the Type 1 piecewise monotonic endomorphism.

[Joint work with Emily Ribando-Gros, Muhammad Abdulla, Naveed Iqbal]

Received: July 28, 2016