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Qualitative Properties of Solutions to Nonlinear Parabolic PDE with Double-Degenerate Fast Diffusion

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Abstract of Report Talk: We consider the Cauchy problem for the double degenerate parabolic partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(\left|\frac{\partial u^m}{\partial x}\right|^{p-1}\frac{\partial u^m}{\partial x}\right) - bu^\beta, x \in \mathbb{R}, t > 0, u(x,0) = C(-x)_+^\alpha \quad (1)$$

where C, α, m, and p are positive constants and $b \in \mathbb{R}$. We prove the existence/non-existence of an interface for this problem and the explicit asymptotics of the solution near the interface or at infinity. The equation arises in a variety of situations including heat transfer in plasma, spatial spread of populations in ecology, diffusion of chemicals through ground-water, etc. Our research is concentrated in the open case of fast diffusion range: $0 < mp < 1$. When $p = 1$, our equation reduces to the nonlinear reaction-diffusion equation with fast diffusion, the properties of which were fully classified in [U.G.Abdulla, Nonlinear Analysis, 50, 4(2002), 541-560]. Through scaling analysis we prove that the behavior of the interface is determined by the competition between diffusion and reaction. We found that if $\beta < mp$, the interface propagates with finite speed; in this case, we prove that the interface expands for $\alpha < \frac{1+p}{mp-\beta}$ and shrinks for $\alpha > \frac{1+p}{mp-\beta}$. In the critical case $\alpha = \frac{1+p}{mp-\beta}$, we prove that there exists a critical value C_* such that the interface expands (or shrinks) if $C > C_*$ (or $C < C_*$). In all of these cases, we rigorously prove explicit bounds for the interface and for the solution u. If $\beta \geq mp$, we prove that the interface expands with infinite speed and we derive explicit asymptotic formulas for the solution at infinity. The rigorous methods we apply include scaling, construction of super- and sub- solutions to the problem, and special comparison theorems in general domains. We also confirmed our results numerically with a WENO scheme and developed test problems for special cases of our equation for a new numerical interface tracking method.

[Joint work with Adam Prinkey]

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