THE YOUNG MATHEMATICIANS CONFERENCE

2015

ABSTRACTS OF PRESENTATIONS

The Ohio State University

The twelfth annual

Young Mathematicians Conference

August 21-23, 2015

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Plenary Talks

In chronological order

Geometry and the Shape of Space

Dave Futer
Temple University Friday, August 21, 5:00 PM

Is our physical universe finite or infinite? Is it interconnected in a way that would allow a (very fast) spaceship to fly off in one direction and come back from another? Does it have handles? These questions from the field of topology can be approached using techniques from the modern geometry of curved spaces. In fact, geometric ideas developed over the last 2 decades give us the first complete list of all possible universes, along with the tools for making a good guess about where our universe fits on the list. I will go over some of the relevant background and explain how these ideas show up in my work.

Patterns of Synchrony: From Animal Gaits to Binocular Rivalry

Marty Golubitsky
The Ohio State University Saturday, August 22, 9:30 AM

This talk will review previous work on quadrupedal gaits and recent work on a generalized model for binocular rivalry proposed by Hugh Wilson. Both applications show how rigid phase-shift synchrony in periodic solutions of coupled systems of differential equations can help understand high level collective behavior in the nervous system.

Wave localization

Svitlana Mayboroda
University of Minnesota Saturday, August 22, 5:10 PM

The talk is devoted to the old question of wave localization, the history of the problem, a thrilling recent mathematical discovery, and multitude of potential applications in science and beyond. Wave localization is an astonishing ability of physical systems to confine vibrations to small portions of their original domains of activity, preventing extended propagation. The context is not restricted solely to mechanical vibrations: light is a particular example of an electromagnetic wave, wi-fi is delivered by waves, sound is a pressure wave, and, from the vantage point of quantum physics, even matter can be perceived as a type of wave. Whether wanted or unwanted, known or ignored, exploited or only sustained, localization plays a significant role in science and technology. The attention globally received by Anderson localization, including the 1977 Nobel prize in physics, provides evidence of this phenomenon's importance. However, for a long time the intricate nature of this phenomenon and perplexing rules that govern it largely remained a mystery. Together with Marcel Filoche, we have recently discovered a new mathematical mechanism describing the spatial confinement of eigenfunctions. Our theory introduces a new concept, the landscape of localization, which, based just on the underlying PDE, determines a partition of the domain into subregions which host localized eigenfunctions, particularly in low energies, and indicates the corresponding frequencies. By now, the power of the landscape has been rigorously proved in our joint work with D. Arnold, G. David, and D. Jerison. It led to the sharp estimates on the exponential decay of low-energy eigenfunctions and the new Weyl-type law. In physics, our theory has been confirmed in experiments with vibrating plates and is about to be used in systems of cold atoms. In engineering, it is exploited in concert with the LED design.

Student Presentations

In alphabetical order by the last name of the first presenter

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On the Fine Classification of Periodic Orbits of Continuous Endomorphisms and Universality in Chaos

Muhammad Abdulla (abdulla@fit.edu)
Alyssa L Turnquist (alt007@morningside.edu)
Florida Institute of Technology [Mentor: Ugur Abdulla]

Abstract of Poster Presentation: We complete the classification of the periodic orbits of period $2^n(2k+1)$, $k > 1$, of the continuous endomorphisms on the real line which are minimal with respect to Sharkovski ordering. By developing the new constructive method suggested recently in *Abdulla et al. J. of Diff. Equat. and Appl., 19,8(2013), 1395-1416,* it is proved that independent of k, there are $2^{2^{n+1}-2}$ types of digraphs (and cyclic permutations) with accuracy up to inverse digraphs. We advance outstanding open problem conjectured in JDEA paper on the structure of the second minimal odd orbits, which are defined as those that immediately follow the minimal orbits with respect to Sharkovski ordering. We pursue full analysis of the second minimal 7-orbits. It is proved that there are 11 types of second minimal 7-orbits with accuracy up to inverses. We apply this result to the problem on the distribution of superstable periodic windows within the chaotic regime of the bifurcation diagram of the one-parameter family of logistic type unimodal maps. It is revealed that by fixing the maximum number of appearances of the periodic windows there is a universal pattern of distribution. For example, by employing the notation n_i for the ith appearance of the n-orbit, all the superstable odd orbits up to 8th appearances while increasing the parameter are distributed according to the universal law

$$\cdots \to (2k+3)_1 \to (2k+9)_5 \to (2k+7)_3 \to (2k+9)_6 \to (2k+5)_2 \to$$
$$\to (2k+9)_7 \to (2k+7)_4 \to (2k+9)_8 \to (2k+1)_1 \to \dots \tag{1}$$

where the branches successfully follow from right to left as $k = 1, 2, ...$ The same universal route is continued to the left for all 8 appearances of the $2^n(2k+1)$-orbits successfully for positive integers $n = 1, 2, ...,$ and with the same order of appearance indices. Every orbit in (1) is universal, in the sense that it has a unique cyclic permutation and digraph independent of the unimodal map. In particular, the first appearance of all the orbits is always a minimal orbit, with precisely Type 1 digraph. It is observed that the second appearance of 7-orbit is a second minimal 7-orbit with Type 1 digraph. The reason for the relevance of the Type 1 minimal orbit is the fact that the topological structure of the unimodal map with single maximum is equivalent to the structure of the Type 1 piecewise linear endomorphism. Yet another important development of this research is the revelation of the pattern of the pattern dynamics with respect to increased number of appearances. Understanding the nature and characteristics of this fascinating universal route is an outstanding open problem for future investigations.

[Joint with Rashad Abdulla]

Normality preserving operations for Cantor series expansions and related fractals

Dylan R Airey (dylan.airey@utexas.edu)
University of North Texas [Mentor: Bill Mance]

Abstract of Report Talk: We say a real number x is normal in base b if each block of k digits occurs with frequency b^{-k} in the b-ary expansion of x. Normal numbers were first introduced

by H. Lebesgue in 1909 and early study of them was done in part by P. Erdös, H. Furstenberg, A. H. Copeland, H. Davenport, and W. Schmidt. D.D. Wall in particular proved that rational multiplication and addition preserve normality in base b. Recently J. Vandehey resolved an open problem of G. Rauzy, showing that transformations of the form $f(x) = \frac{ax+b}{cx+d}$ with $ad - bc \neq 0$ preserve normality with respect to the continued fraction expansion. We study related normality preserving operations for the Q-Cantor series expansions. Q-Cantor series expansions are a generalization of b-ary expansions first studied by G. Cantor. Instead of a single base, we define an expansion of a real number x with respect to a basic sequence $Q = (q_n)$ where $q_n \geq 2$ is an integer as follows

$$x = E_0 + \sum_{i=1}^{\infty} \frac{E_i}{q_1 q_2 \cdots q_i}$$

where $0 \leq E_i \leq q_i - 1$ and $E_i \neq q_i - 1$ infinitely often. We study operations which preserve two different notions of normality with respect to Q-Cantor series expansions: Q-normality which is based on asymptotic frequencies of blocks of digits and Q-distribution normality which is based on the equidistribution of orbits. In particular, while it is simple to show integer multiplication preserves Q-distribution normality, we proved that it fails to preserve Q-normality in a particularly strong manner. We also proved that Q-distribution normality is not preserved by non-integer rational multiplication on a set of zero measure and full Hausdorff dimension.

Spaces of Nondegenerate Hypermatrices

Colin C Aitken (caitken@mit.edu)
University of Minnesota [Mentor: Joel Lewis]

Abstract of Report Talk: In this talk, we construct a useful extension of Gaussian elimination to show that if F is a topological field, then there is a transitive, free, and continuous action of a natural quotient of $GL_k(F) \times GL_{k+1}(F)$ on the set $M_k(F)$ of $2 \times k \times (k+1)$ hypermatrices over F with nonzero hyperdeterminant.

The hyperdeterminant is a natural extension of the determinant to arrays of field elements in more than two dimensions. In general, it is difficult to compute, and even the problem of checking whether it is zero has been shown by Hillar and Lim to be NP-hard for general three-dimensional arrays.

We use this action to answer a number of relevant questions, including determining the homotopy groups of $M_k(\mathbb{C})$, counting elements of $M_k(\mathbb{F}_q)$ (generalizing a conjecture of Lewis and Sam), and computing hyperdeterminants for $2 \times k \times (k+1)$ hypermatrices in $O(k^4)$ time, which we use to generalize a formula of Bremner for the $2 \times 2 \times 3$ hyperdeterminant.

Existence of k-Normal Elements in Finite Field Extensions

Loren J Anderson (loren.james.anderson@ndsu.edu)
Pennsylvania State University [Mentor: Gary Mullen]

Abstract of Report Talk: The Primitive Normal Basis Theorem is a powerful result in finite field theory, as it guarantees the existence of a primitive normal element in every field extension. These elements have been well studied due to their applications in areas such as cryptography. Huczynska, Mullen, Panario, and Thomson [2013] extended the definition of normal elements: an element $\alpha \in \mathbb{F}_{p^n}$ is *k-normal* (or has *normalcy k*) over $\mathbb{F}_p$ if $gcd\left(\sum_{i=0}^{n-1} \alpha^{p^i} x^{n-1-i}, x^n - 1\right)$ has degree k. By this definition, normal elements are 0-normal. They also posed an open problem of determining the existence of elements "nearest" to primitive normal elements in the sense of having high order and low normalcy.

We begin by proving that multiplying a k-normal element with an $(n-1)$-normal element in $\mathbb{F}_{p^n}$ over $\mathbb{F}_p$ yields another k-normal element. We then determine the field extensions $\mathbb{F}_{p^n}$ over $\mathbb{F}_p$ in which $\mathbb{F}_{p^n}^*$ can be partitioned into two sets: $(n-1)$-normal elements and elements having order relatively prime to $p-1$. This allows us to answer the open problem for 0-normal and 1-normal elements of order $\frac{p^n-1}{2}$ for a certain class of extensions. Our proof is dependent on the Primitive Normal Basis Theorem and an existence result of Huczynska, Mullen, Panario, and Thomson concerning primitive 1-normal elements. Some insights into determining explicit formulas for the number of elements of arbitrary order and normalcy will be discussed, especially in the notable case of primitive normal elements.

Biases in Second Moments of Satake Parameters of L-Functions

Megumi A Asada (maa2@williams.edu)
Williams College [Mentor: Steven Miller]

Abstract of Report Talk: L-functions occupy a central role in modern number theory, encoding the solution to numerous arithmetically important questions, ranging from the distribution of the number of primes to the structures of the group of rational solutions for elliptic curves to the size of the class number. These L-functions can be defined as an infinite product of local factors, which are themselves products of the Satake parameters:

$$L(s,\phi) = \prod_p \prod_{j=1}^{n} (1 - \alpha_{\pi;i}(p)p^{-s})^{-1}.$$

The moments of a given L-function are sums of powers of Satake parameters; knowing these moments is equivalent to understanding fine properties of the L-function. In previous work the universality of the first and second moments led to powerful connections between number theory and random matrix theory, specifically between the spacings between zeros of L-functions and energy levels of heavy nuclei.

Recently, it was observed that the second moments of the Satake parameters exhibit a bias in many families of of elliptic curve L-functions, and it was conjectured that similar biases exist in other families, which will control the rate of convergence of many arithmetic quantities. We resolve the conjecture for families of Dirichlet L-functions, cuspidal newforms, and holomorphic families of automorphic forms on GL(2) by exploiting powerful trace formulas and orthogonality relations.

[Joint with Eva Fourakis, Kevin Yang]

Visualizing Dessins d'Enfants on the Torus

Leonardo R Azopardo (lazopard@purdue.edu)
Sarah E Thomaz (sthomaz@purdue.edu)
Purdue University [Mentor: Edray Goins]

Abstract of Poster Presentation: A Belyĭ map $\beta : \mathbb{P}^1(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$ is a rational function with at most three critical values; we may assume these values are $\{0, 1, \infty\}$. A Dessin d'Enfant is a planar bipartite graph obtained by considering the preimage of a path between two of these critical values, usually taken to be the line segment from 0 to 1. Replacing $\mathbb{P}^1$ with an elliptic curve E, there is a similar definition of a Belyĭ map $\beta : E(\mathbb{C}) \to \mathbb{P}^1(\mathbb{C})$. The corresponding Dessin d'Enfant can be drawn on the torus by composing with an elliptic logarithm: $\beta^{-1}\big([0,1]\big) \subseteq E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$.

In this project, we use the open source Sage to write code which takes an elliptic curve E and a Belyĭ map β to return a Dessin d'Enfant on the torus in two and three dimensions. Following a 2013 paper by Cremona and Thongjunthug, we make the elliptic logarithm

$E(\mathbb{C}) \simeq \mathbb{C}/\Lambda$ explicit using a modification of the arithmetic-geometric mean, then compose with a canonical one-to-one correspondences $\mathbb{C}/\Lambda \simeq \mathbb{T}^2(\mathbb{R})$.

[Joint with Avi Steiner]

Properties of m^{th}-Level Triangle Numbers in Second-Order Recursive Polynomials

Maximillian C.W. Bender (mbender1@conncoll.edu)
Maria I. Lema (lemami01@mail.buffalostate.edu)
Alperen Sirin (saidsirin@gmail.com)
Michigan State University [Mentor: Aklilu Zeleke]

Abstract of Report Talk: Through the application of Budan's Theorem, which relates the number of roots of a polynomial in a given interval to the sign changes in the sequence of coefficients of that polynomial, we present analytic and combinatorial properties of a generalized sequence of recursive polynomials. Here, we follow the standard initial conditions for a recursive polynomial $G_n(x)$ with $G_0(x) = -1$ and $G_1(x) = x - 1$. We consider the generalized recurrence relation $G_n(x) = x^k G_{n-1}(x) + x^\ell G_{n-2}(x)$, where ℓ and k are natural numbers. Our results include properties of the roots, the growth of the sequence of derivatives, and the general structure of these polynomials. In particular, we prove that the maximal roots of the polynomials generated in the case where $\ell = k$ converge to 2 and all nonzero roots of $G_n(x)$ for any k are irrational for $n > 2$. We also discuss the relevance of the triangle numbers in the formation of these polynomials and present a game theoretical application along with modular identities evaluating $G_n(x)$ modulo $G_1(x)$ and $G_2(x)$.

Discrete Morse Theory and the Rational Homology of Thompson's Group T

Fernando E Betancourt Velez (fernandobetancourt1993@gmail.com)
Angelo N Taranto (ant9@njit.edu)
Miami University [Mentor: Daniel Farley]

Abstract of Report Talk: Thompson's group T was introduced by Richard Thompson in 1965 and is an infinite simple group with a finite presentation. The homology of T was first calculated by Ghys and Sergiescu in 1987 using very complicated tools. We attempt to compute the homology of T directly from the cellular chain complex $\mathcal{C}$ associated to a natural cell complex on which T acts. To simplify calculations, we use discrete Morse theory on $\mathcal{C}$ to create a more manageable object called the Morse complex. More specifically, we construct a discrete gradient vector field on the complex $\mathcal{C}$ and use a filtration by sub-complexes to create finitely generated chain groups with which to calculate the homology.

As a result of our methods, we are able to compute up to the second homology by hand and we are in the process of determining the generators for the cohomology ring of T on the cochain level.

[Joint with Sabrina Lato]

Sign Characteristic Preserving Linearizations in $DL(P)$.

Joseph J Breen (josephbreen2016@u.northwestern.edu)
Sarafina I Ford (sif@umail.ucsb.edu)
UC, Santa Barbara [Mentor: Maribel Bueno]

Abstract of Report Talk: A matrix polynomial $P(\lambda)$ is a polynomial with matrix coefficients. Due to their presence in areas such as control theory and signal processing, the computation

of eigenvalues and eigenvectors of matrix polynomials is an important and difficult problem. The standard approach is to use linearizations, which are matrix polynomials of degree 1 that share the eigenvalues of $P(\lambda)$.

Hermitian matrix polynomials and their real eigenvalues are of particular interest in applications. Attached to these eigenvalues is a set of signs called the sign characteristic, which establishes a relationship between the left and right eigenspaces of $P(\lambda)$. Thus, it is desirable to find linearizations that preserve this sign characteristic.

In our work, we consider the vector space $\mathbb{DL}(P)$ of block-symmetric linear matrix polynomials, almost all of which are linearizations of $P(\lambda)$, constructed in 2006 by Mackey, Mackey, Mehl, and Merhmann. Using techniques from indefinite linear algebra, we give a full characterization of the linearizations in $\mathbb{DL}(P)$ that preserve the sign characteristic of $P(\lambda)$. Strikingly, the sign characteristic of a linearization depends only on the sign characteristic of $P(\lambda)$ and the sign of a polynomial associated with the linearization when evaluated on the real spectrum of $P(\lambda)$. Furthermore, we give a procedure for generating large classes of sign-characteristic preserving linearizations in $\mathbb{DL}(P)$ that only requires computing the norms of the coefficients of $P(\lambda)$.

[Joint with Susana Furtado]

An Unoriented Variation on de Bruijn Sequences

Christie Burris (christie.burris@gmail.com)
Colorado State University [Mentor: Patrick Shipman]

Abstract of Report Talk: For positive integers k, n, a de Bruijn sequence $B(k, n)$ is a finite sequence of elements drawn from k characters whose subwords of length n are exactly the k^n words of length n on k characters. This paper introduces the unoriented de Bruijn sequence $uB(k, n)$, an analog to de Bruijn sequences, but for which the sequence is read both forwards and backwards to determine the set of subwords of length n. We show that nontrivial unoriented de Bruijn sequences of optimal length exist if and only if k is two or odd and n is less than or equal to 3. Unoriented de Bruijn sequences for any k, n may be constructed from certain Eulerian paths in Eulerizations of unoriented de Bruijn graphs.

[Joint with Francis Motta]

A Twisted Second Moment for Automorphic L-Functions

David R. Burt (drb3@williams.edu)
Blaine Talbut (blainetalbut@uchicago.edu)
Williams College [Mentor: Steven Miller]

Abstract of Report Talk: As L-functions (generalizations of the famous Riemann zeta function) play a central role in many problems in modern number theory, it is desirable to understand their behavior. Similar to how moments of a probability density provide information about its shape, many properties of L-function can be determined from knowledge of its moments. Current techniques, however, are strong enough only to obtain fourth moments for GL(1) L-functions, second moments for GL(2) L-functions, and averages for higher-order families. In 2008, Hughes and Young obtained the fourth moment for the Riemann zeta function multiplied ("twisted") by an arbitrary Dirichlet polynomial. In 2011, Young obtained the same result for general Dirichlet L-functions. In 2015, Balkanova obtained an average fourth moment for automorphic L-functions of prime power level.

We further develop these techniques and are hopeful that we will be able to determine the average twisted second moments over the family of automorphic L-functions, i.e., automorphic L-functions multiplied by an arbitrary Dirichlet polynomial; the freedom to vary the twisting polynomial is essential in many applications, such as studying gaps between zeros on the critical line. We introduce arithmetic weights $Z(1, f)$ which facilitate applying the

Petersson formula to average over the family. That is, we evaluate

$$\sum_{f\in H_k^*(q)} \frac{1}{Z(1,f)} \int_{-\infty}^{\infty} |M(\tfrac{1}{2}+it)|^2 L(\tfrac{1}{2}+\alpha+it,f)L(\tfrac{1}{2}+\beta-it,\hat{f})w(t)dt,$$

where $H_k^*(q)$ is the space of cusp forms of weight k and level q, M is our arbitrary twisting polynomial, and α and β are our shifts. Our preliminary investigations are promising for prime level q; we have reduced the problem to sums involving Kloosterman sums and Bessel functions, which we are currently analyzing using Dirichlet characters and results on special functions.

[Joint with Owen Barrett, Caroline Turnage-Butterbaugh]

Evolution of Free Boundaries for the Nonlinear Fokker-Planck Equation

Christie M Campbell (campb744@msu.edu)
Luke T Andrejek (landreje@indiana.edu)
Florida Institute of Technology [Mentor: Ugur Abdulla]

Abstract of Report Talk: We investigate the problem on short-time behavior of free boundaries and local solutions near it in the following Cauchy problem for the nonlinear degenerate diffusion equation with convection:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u^m}{\partial x^2} + b\frac{\partial u^\gamma}{\partial x}, x\in\mathbb{R}, t>0; u|_{t=0} = C(-x)_+^\alpha \tag{2}$$

where $m > 1, \gamma > 0$. The PDE in (1) is called the nonlinear Fokker-Planck equation. It arises in many applications in physics, biology, and chemistry; examples include infiltration of water in a homogeneous porous medium, transport of thermal energy in plasma, spatial transport of populations, etc. The problem of determining the short-time behavior of the free boundaries, or interfaces, is known as the Barenblatt problem. Full solution of this problem for the reaction-diffusion equation was given in [Abdulla and King, SIAM J. Math. Anal., 32, 2(2000), 235-260] and [Abdulla, Nonlinear Analysis, 50, 4(2002), 541-560], but the problem is open for reaction-diffusion-convection equations. The goal of this project is to apply the methods of these papers to solve the open problem for the diffusion-convection equation (1).

It is proved that the behavior of the interface in the case $b > 0$ depends on the outcome of the competition between two opposing factors: diffusion and convection. By applying the scaling method we identify various regions of the (α,γ)-parameter space where one of the factors dominate over the other. It is proved that if either $0 < \gamma < \frac{m+1}{2}, 0 < \alpha < \frac{1}{m-\gamma}$ or $\gamma \geq \frac{m+1}{2}, 0 < \alpha < \frac{2}{m-1}$, then diffusion dominates and the interface expands. If $\alpha > \frac{1}{m-\gamma}$ then convection dominates and the interface either shrinks or remains stationary. On the borderline case, when $\alpha = \frac{1}{m-\gamma}, 0 < \gamma < \frac{m+1}{2}$, the interface may initially expand, shrink or remain stationary, depending on the parameter C. It is proved that there is a critical $C*$ determined by the parameters of the problem such that for $C < C*$ and $C > C*$, the interface shrinks and expands respectively, while for $C = C*$, the solution is stationary. In all cases, we prove explicit formulae for the interface and local solution, with precise estimations up to constant coefficients. The rigorous proof methods that we use are rescaling, construction of super- and subsolutions and special comparison theorems in irregular domains. A WENO scheme was applied to the problem and the numerical results support our proved estimations.

[Joint with Adam Prinkey, Jonathan Goldfarb]

Convergence Preserving Permutations and Divergent Fourier Series

Angel D Castillo (angelcastillo15@tamu.edu)
Jose Chavez (JoseChavez5@my.unt.edu)
University of Michigan-Dearborn [Mentor: Yunus Zeytuncu]

Abstract of Report Talk: The Fourier series of a continuous function f on $[-\pi, \pi]$ is an infinite sum of trigonometric functions that approximates f, in some sense. During the mid 19^{th} century, it was believed that the Fourier series of a continuous function would converge pointwise back to the function. However, this is not always the case. In 1911 Fejér introduced a counterexample of a continuous function $\mathbf{F}(\mathbf{x})$ whose Fourier series diverges at a point. Almost 100 years later, McNeal and Zeytuncu proved that using a specific convergence preserving permutation, the terms of the Fourier series of $\mathbf{F}(\mathbf{x})$ could be rearranged to converge pointwise.

Our study focuses on understanding which divergent Fourier series can be made to converge by convergence preserving permutations. In particular, we prove that not every divergent Fourier series can be rearranged to converge by a convergence preserving permutation. By combining ideas from Fejér original construction and recent observations on convergence preserving permutations by Velleman, we present an explicit function $\mathbf{G}(\mathbf{x})$ with the following properties:

1.) $\mathbf{G}(\mathbf{x})$ is continuous on $[-\pi, \pi]$.

2.) The Fourier series of $\mathbf{G}(\mathbf{x})$ diverges at $x = 0$.

3.) This divergence *cannot* be rearranged to converge by *any* convergence preserving permutation. However, it can be made to converge by a non-convergence preserving permutation.

Finally, we show a partial classification of continuous functions whose Fourier series exhibits this behavior.

Upper Bounds for Stern's Diatomic Sequence and Related Sequences

Colin R Defant (cdefant@ufl.edu)
University of Florida

Abstract of Report Talk: Let $(s_2(n))_{n=0}^{\infty}$ denote Stern's diatomic sequence. For $n \geq 2$, we may view $s_2(n)$ as the number of partitions of $n-1$ into powers of 2 with each part occurring at most twice. More generally, for integers $b, n \geq 2$, let $s_b(n)$ denote the number of partitions of $n-1$ into powers of b with each part occurring at most b times. Using this combinatorial interpretation of the sequences $s_b(n)$, we use the transfer-matrix method to develop a means of calculating $s_b(n)$ for certain values of n. This then allows us to derive upper bounds for $s_b(n)$. In the special case $b = 2$, our bounds improve upon the current upper bounds for the Stern sequence. In 1982, Berlekamp, Conway, and Guy asked for the exact value of $\limsup\limits_{n\to\infty} \dfrac{s_2(n)}{n^{\log_2 \phi}}$. Calkin and Wilf later showed that this value lies in the interval $[3^{\log_2 \phi}/\sqrt{5}, (1+\phi)/2]$, where ϕ is the golden ratio. It was not until 2014 that Coons and Tyler were able to show that the value of this supremum limit is in fact $3^{\log_2 \phi}/\sqrt{5}$. Here, we are able to use our upper bounds for the sequences $s_b(n)$ to show that, in general,

$$\limsup_{n\to\infty} \frac{s_b(n)}{n^{\log_b \phi}} = \frac{(b^2-1)^{\log_b \phi}}{\sqrt{5}}.$$

Characterizing the Dynamics of Certain Sequential Dynamical Systems

Colin R Defant (cdefant@ufl.edu)
UC Santa Barbara [Mentor: Padraic Bartlett]

Abstract of Poster Presentation: A sequential dynamical system (SDS) is a dynamical system defined over a graph in which vertices update sequentially. We describe a method, reliant on the concept of topological conjugacy, for enumerating the periodic points of certain SDS. Namely, we choose an integer $n \geq 3$ and consider the SDS defined over C_n, the cycle graph on n vertices, using an identity update order. We either use the update rule parity or the update rule $1 +$ parity. We let $\alpha_n(r)$ denote the number of periodic points of period r of the SDS defined using parity. Similarly, we let $\delta_n(r)$ denote the number of periodic points of period r of the SDS defined using $1 +$ parity. Using Möbius inversion, we give explicit formulas for $\alpha_n(r)$ and $\delta_n(r)$. As a surprising consequence of these formulas, we find that if we fix r and vary n, there are only two possible nonzero values of $\alpha_n(r)$ and only one possible nonzero value of $\delta_n(r)$.

One-Level Density for Cusp Forms of Arbitrary Level

Jonathan DeWitt (jdewitt@haverford.edu)
Williams College [Mentor: Steven Miller]

Abstract of Report Talk: The Katz-Sarnak philosophy states that matrices from the classical compact matrix groups serve as good models for the properties of the zeros of L-functions. Let $H_k^\star(N)$ be the set of holomorphic cusp forms of weight k which are new of level N. In their seminal paper, Luo, Iwaniec and Sarnak studied the 1-level density of the zeros of holomorphic cusp forms of square-free level and, assuming the Riemann hypothesis for the arising L-functions, established the main term of the limiting densities as suggested by the Katz-Sarnak Philosophy agree with Random Matrix Theory. Recently Rouymi proved analogous results valid for prime-power level. By using the basis for the space of holomorphic cusp forms recently obtained by Blomer and Milićević, we obtain trace formulas valid for arbitrary level and prove the limiting densities exist for arbitrary level and agree with Random Matrix Theory. There are numerous obstructions to this work due to the immense simplifications which are obtained when the level is square-free or a prime power (including numerous mistakes in the literature which have been corrected).

[Joint with Owen Barrett, Paula Burkhardt, Robert Dorward]

Limiting Spectral Measures for Random Checkerboard Matrices

Jonathan DeWitt (jdewitt@haverford.edu)
Williams College [Mentor: Miller Steven]

Abstract of Poster Presentation: Random matrix theory has important applications to physics as well as number theory through the Hilbert-Polya conjecture, which states that the non-trivial zeros of the Riemann zeta function correspond to the eigenvalues of some self-adjoint operator. In particular, patterned ensembles of matrices such as Toeplitz matrices and Circulant matrices have been studied, as well as the less structured classical ensemble of Wigner matrices. We investigate a modified Wigner ensemble, the k-checkerboard ensemble, in which we set all of the entries a_{ij} of our random matrix to a constant value when $i \equiv j \bmod k$. The matrices in this ensemble contain two groups of eigenvalues. The first (comprising most of the spectrum) are of order $\sqrt{n}$ and the second are of order n. This ensemble presents major obstacles to the application of standard techniques, such Markov's method of moments, as the average moments diverge. Building on a result of Tao, we prove that with $\sqrt{n}$ normalization the k-checkerboard matrices converge almost surely to the semi-circle distribution, despite the obstructions to the method of moments. Then by using a non-standard normalization,

we obtain almost sure convergence to a limiting distribution for the eigenvalues of order n as well. In all cases we determine the effect of varying k on the resulting limiting distribution.

[Joint with Paula Burkhardt, Kevin Yang]

New Linear Codes from Quasi-twisted Codes

Derek S Foret (foretd@kenyon.edu)
Kenyon College [Mentor: Nuh Aydin]

Abstract of Poster Presentation: Constructing codes with the best possible parameters is one of the fundamental and challenging problems in coding theory. There are well-known databases of best-known codes over finite fields of sizes up to nine. Additionally, new databases of best known codes over the alphabets GF(11) and GF(13) have been recently introduced. Due to computational complexity of computing the minimum distance of an arbitrary linear code, researchers focus on codes with rich mathematical structures. Quasi-twisted (QT) codes (and their special subset constacyclic codes) are a promising class of linear codes that have been shown to contain many record-breaking codes, that is, codes with better parameters than the best-known codes in the databases mentioned above. Continuing on the work by J. M. Murphree, who presented his research at YMC 2013, we use his results on the best (and new) constacyclic codes over GF(3), and use them to construct new QT codes over GF(3). Additionally, we have found new linear codes over GF(11) and GF(13) from constacyclic and QT codes. Building on earlier research, we used a comprehensive search strategy that avoids redundantly checking constants of a finite field that belong the same conjugacy class. Whenever possible we conducted exhaustive searches. When that was not possible, we examined a large subset of the search space. Our search yielded new record-breaking codes over the finite fields GF(3), GF(11) and GF(13).

Geometric invariants of complex polynomials

Weibo Fu (fuweibo1994@gmail.com)
University of Michigan [Mentor: Michael Zieve]

Abstract of Report Talk: Two of the most important geometric invariants of a complex polynomial are its monodromy group and its ramification type. In topology, a fundamental theorem by René Thom describes all possibilities for the ramification type of a complex polynomial. In Galois theory, a much-used theorem by Walter Feit and Peter Müller describes all possibilities for the monodromy group of an indecomposable complex polynomial. Here we say that a polynomial is *indecomposable* if it cannot be written as the functional composition of lower-degree polynomials; thus, indecomposable polynomials are the “prime” polynomials under composition, and it turns out that many questions about arbitrary polynomials can be resolved if they can be solved for indecomposable polynomials. We prove a result which brings together the results of Thom and Feit–Müller by determining all possibilities for the pair (monodromy group of f, ramification type of f) where f varies over all indecomposable complex polynomials. Our proof involves a delicate reduction to the case of polynomials with only two (finite) critical values, a reinterpretation and resolution of the latter case in terms of a question about bicolored trees, and methods from group theory and Galois theory. Beyond the intrinsic importance of determining all possibilities for the fundamental invariants of an indecomposable polynomial, our result provides a new tool which helps resolve many questions about polynomials arising in dynamical systems, complex analysis, number theory, algebraic geometry, and other areas.

A Finiteness Property of Torus Invariants

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Abstract of Poster Presentation: In this REU project the invariant subring R_n of an algebraic torus $T = (\mathbb{C}^\times)^r$ acting on the multi-homogenous polynomial ring

$$S^{\boxtimes n} = \bigoplus_{d=0}^{\infty} (S^{(d)})^{\otimes n},$$

where $S^{(d)}$ is the dth graded piece of the polynomial ring $S = \mathbb{C}[x_1, \ldots, x_k]$, is studied from the viewpoint of matrices whose entries sum to zero. Using these weight matrices we prove that there exists a δ_1 such that for all positive integers n, the relations of the invariant subring R_n are generated in multi-homogenous degree $\leq \delta_1$.

[Joint with Samuel Tenka]

Point Sets Determining Each Distinct Distance a Unique Number of Times

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Sarah E Manski (sarah.manski12@kzoo.edu)
Williams College [Mentor: Eyvindur Palsson]

Abstract of Poster Presentation: In 1989, Erdős conjectured that for a sufficiently large n it is impossible to place n points in general position in a plane so that they determine exactly $n-1$ distinct distances, with each distance appearing a unique number of times (the points are required to be in general position as otherwise an arithmetic progression on the line trivially resolves the problem). In his initial paper, Erdős provided constructions for $n \leq 6$. In the same year, Palásti came up with constructions for $n \leq 8$. Constructions for $n = 9$ and above remain undiscovered, and little headway has been made toward a proof that for sufficiently large n no configuration exists. By analyzing related problems in number theory, combinatorics and geometry we come up with new constructions of points satisfying this condition in the plane, as well as the natural generalization to higher dimensions. We also show that for any given n there exists a sufficiently large dimension d such that there is a configuration in d-dimensional space meeting Erdős' criteria.

[Joint with David Burt, Steven Miller, Hong Suh]

Sharpness Of Falconer's Incidence Theorem In Higher Dimensions

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Williams College [Mentor: Eyvindur Palsson]

Abstract of Report Talk: Falconer's distance conjecture relating Hausdorff dimension to Lebesgue measure is one of the major open problems in geometric measure theory. Efforts to study this problem, as well as its generalizations from distances to k-simplices, have used Falconer's incidence theorem; however, there is a relative dearth of constructions demonstrating the tightness of higher dimensional versions of Falconer's generalized incidence theorem. We discuss our new constructions, which improve the best bounds. By applying the results of the Gauss circle problem, we obtain improved bounds for k-simplices in d-dimensional space using a lattice construction. Furthermore, we generalize an example of Mattila raising the lower bound on dimension to $\frac{d+1}{2}$ when $k \leq d \leq 2k+1$. Finally, by studying an example of Valtr related to the Erdős repeated distance problem, we obtain the bound $\frac{d+1}{2}$ for 2-simplices in every dimension using the norm induced by a convex body, which improves on

the best known result. The study of this final result utilizes number-theoretic machinery to estimate the number of solutions to a diophantine equation.

[Joint with Steven Miller, Gwyneth Moreland]

Branching types of rational functions and functional decomposition

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Heyi Zhu (hzhu@haverford.edu)
University of Michigan [Mentor: Michael Zieve]

Abstract of Report Talk: In 1891, Hurwitz showed that there are only finitely many topologically inequivalent maps $S^2 \to S^2$ induced by rational functions $f(x)$ of a prescribed degree n. He showed further that each topological equivalence class corresponds to a finite collection of partitions of n, known as the *branching type* of f. In the ensuing 124 years, the *Hurwitz problem* of describing which branching types actually occur has become a fundamental problem in topology. We introduce a new approach to this problem, by showing that if the branching type of f belongs to one of several explicit infinite families, then there must be a functional decomposition $f = f_1 \circ f_2$ where f_1 and f_2 are lower-degree rational functions such that the branching type of f_1 is explicitly known. This is useful because, for several of the branching types of f that we consider, it is easy to show that they could never occur as the branching type of such a composition $f_1 \circ f_2$, and hence that they do not occur as the branching type of any rational function $f(x)$. Zheng has shown that there are 27176 batches of partitions of a number $n \leq 22$ which satisfy the known necessary conditions for being the branching type of a degree-n rational function but which do not actually occur as such a branching type. Our results provide a new necessary condition which explains over 99% of these non-occurring types. As such, it seems that our results pinpoint the main remaining obstruction to the occurrence of a branching type, and therefore bring us much closer to the final resolution of the Hurwitz problem.

[Joint with Ben Myers, Danny Neftin]

Ranks of permutative matrices

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College of William and Mary [Mentor: Charles Johnson]

Abstract of Poster Presentation: By a (symbolic) permutative matrix, we mean an m-by-n matrix whose entries are chosen from among n distinct variables in such a way that each row is a different permutation of the variables. Though the concept is new, it generalizes Latin squares, relates to the nonnegative inverse eigenvalue problem and has arisen in special situations in prior papers. We are primarily interested in the ranks that can occur among distinct positive substitutions for the variables. These can be identically invertible, identically singular, or sometimes invertible.

We identify two different ways that identical singularity can occur: **row grouping** and **h, k-partitions**, as well as variants upon these. All are combinatorial in nature. An h, k-partition is a partition of the rows into h parts and the columns into k parts, in such a way that each block has as many variables appearing in it as the number of columns. The rank is then bounded by $n + h - k$. Corresponding proofs are available and a number of particular results are included. An algorithm to find row grouping and h, k-partition in a permutative matrix will also be presented.

[Joint with Caroline Davis, Yimeng Zhang]

Isoperimetric Inequality in the Third Dimension

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Abstract of Poster Presentation: The great importance of the isoperimetric constant in 3D, is due to its multiple applications in the real world. The ratio of surface area-to-volume is especially relevant to biology and chemistry. Although there exist many proofs for the isoperimetric inequality in 2D, such as Steiner's geometric argument and Schmidt's analytic one, there are far fewer proofs for the three dimensional case. Those that exist are beyond the scope of a calculus course. Thus, we attempt to develop a more elementary proof for the 3D isoperimetric inequality that reads,

$$\frac{(SA)^{3/2}}{V} \geq 6\sqrt{\pi}.$$

In particular, we focus on the special case of solids of revolution. For a given solid of revolution, there are integral formulas for surface area and volume. Therefore, if $f(t)$ is a differentiable function on an interval $[a, b]$ that is positive on (a, b), and 0 at a and b, then the isoperimetric inequality can be written as an integral inequality as

$$\frac{(SA)^{3/2}}{V} = \frac{\left(2\pi \int_a^b f(t)\sqrt{1+(f'(t))^2}dt\right)^{3/2}}{\pi \int_a^b (f(t))^2 dt} \geq 6\sqrt{\pi}.$$

We indeed present an elementary proof of this estimate when we add the extra assumption that the surface area is sufficiently large. The main ingredients of our proof are integration-by-parts and the geometric/arithmetic mean inequality.

[Joint with Hyejin Kim]

The Relationship Between a Local Ring and Its Completion

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Williams College [Mentor: Susan Loepp]

Abstract of Report Talk: A local ring R is a Noetherian ring with exactly one maximal ideal. We can define a metric on the ring using this maximal ideal, and complete it as a metric space; the completion $\hat{R}$ of a local ring is local as well. The theory of complete local rings is rich, and so we would like to better understand the relationship between a local ring and its completion in order to see what nice properties of $\hat{R}$ also hold for R. In particular, we study the formal fibers of prime ideals of R.

The formal fiber of R at a prime ideal $\mathfrak{p}$ consists of the prime ideals $\mathfrak{q}$ of $\hat{R}$ such that $\mathfrak{q} \cap R = \mathfrak{p}$. The dimension of the formal fiber of R at $\mathfrak{p}$ is denoted $\alpha(R, \mathfrak{p})$, and is defined to be the maximal length of chains of its prime ideals. For most rings one encounters, such as a polynomial ring in n variables over a field, the formal fiber of (0) has dimension $n-1$, and larger prime ideals have lower dimensional formal fibers. Heinzer, Rotthaus, and Sally have conjectured that for a local domain R with $\alpha(R, (0)) > 0$, the set of height 1 primes $\mathfrak{p}$ satisfying $\alpha(R, \mathfrak{p}) = \alpha(R, (0))$ is finite, where a height 1 prime ideal is one that contains no other nonzero prime ideals. In this talk, we prove that this set can in fact be infinite by constructing a unique factorization domain with the unusual property that $\alpha(R, (0)) = \alpha(R, \mathfrak{p})$ for every prime ideal $\mathfrak{p}$ of height 1. Given a complete local UFD T, we adjoin specific transcendental elements to the subring generated by 1 and successively build up larger subrings. Using transfinite induction, we see that the constructed ring has completion T and has the desired properties.

[Joint with Sarah Fleming, Peter McDonald, Nina Pande, David Schwein]

Theoretical Friends of Finite Proximity

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Auburn University [Mentor: Peter Johnson]

Abstract of Report Talk: For a positive integer n, the abundancy index of n is $I(n) = \frac{\sigma(n)}{n}$, where $\sigma(n)$ is the sum of the positive divisors of n. Two distinct positive integers are friends if they have the same abundancy index. A theoretical friend of proximity t of a positive integer m is a sequence $s = \{s_k\}_{k=1}^{\infty}$ of positive integers such that $\lim_{k\to\infty} I(s_k) = I(m)$; $|P_s| = t$, where $P_s = \{p : p \text{ is a positive prime and } p \text{ divides } s_k$ for some $k\}$; and $s_k \neq m$ for all k. We say that s is a theoretical friend of finite proximity (TFOFP) of a positive integer m if and only if s is a theoretical friend of m of proximity t for some positive integer t.

In 2008, Jeffrey Ward posed the question: Does every positive integer have a TFOFP? Using some properties of the abundancy index, we show that the answer to this question is no. For example, 1 does not have a TFOFP. By taking subsequences of subsequences, we show that if a positive integer has a TFOFP then it must have a TFOFP of a certain form. Using this form, we show that no odd prime has a TFOFP and certain integers have a TFOFP if and only if they have a friend.

[Joint with John Ryan]

General State-Sum Construction of Topological Quantum Field Theories with Defects

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Gail K Jardine (gkjardine@byu.edu)
Kansas State University [Mentor: David Yetter]

Abstract of Report Talk: We derive the general state sum construction of two-dimensional topological quantum field theories (2-D TQFTs) with source defects on oriented curves, extending the state-sum construction from special symmetric Frobenius algebra for 2-D TQFTs without defects (cf. Lauda & Pfeiffer [2007]). From the extended Pachner moves (Crane & Yetter [2014]), we derive equations that we subsequently translate into string diagrams so that we can easily observe their properties. As in Dougherty, Park and Yetter [2014], we require that triangulations be flag-like, meaning that each simplex of the triangulation is either disjoint from the defect curve, or intersects it in a closed face, and that the extended Pachner moves preserve flag-likeness.

This research was conducted under the mentorship of Prof. David Yetter at Kansas State University with the support of NSF grant DMS-1262877.

An explicit solution formula for the discrete Schrödinger equation on a half line lattice

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The University of Texas at Arlington [Mentor: Tuncay Aktosun]

Abstract of Poster Presentation: The discrete Schrödinger equation

$$-\psi(n+1) + 2\,\psi(n) - \psi(n-1) + V(n)\,\psi(n) = \lambda\,\psi(n), \qquad n \geq 1,$$

is a second-order difference equation used to describe the quantum mechanical behavior of a particle of energy λ in the force field of a real-valued potential $V(n)$. The goal is to obtain a large class of explicit solutions to this difference equation in a systematic way in terms of elementary functions of the discrete variable n. This is done in two stages. In the first stage the direct scattering problem is analyzed, and in the second stage the inverse scattering

problem is analyzed. In the direct problem, given potential values $V(n)$ the corresponding scattering matrix is determined. In the inverse problem, given the scattering matrix the corresponding potential values are determined. An explicit solution formula is developed by choosing the scattering matrix as a rational function of the parameter z, where λ and z are related as $\lambda = 2 - z - z^{-1}$. The same problem is also viewed as the discrete Schrödinger equation on the full line lattice by using $V(n) = 0$ when $n \leq 0$. The direct scattering problem is then analyzed by evaluating the corresponding transmission and left reflection coefficients $T(z)$ and $L(z)$, respectively. When $T(z)$ and $L(z)$ are rational functions of z, by solving the inverse problem an explicit solution formula is developed for the discrete Schrödinger equation.

Counting 10-Arcs in the Projective Plane over Finite Fields

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Yale University [Mentor: Nathan Kaplan]

Abstract of Report Talk: An n-arc in the projective plane over a finite field is a collection of n distinct points in the plane, no three of which lie on a line. Formulae for the number of n-arcs are known only through $n = 9$; we seek an analogous formula for 10-arcs. Building on the work of D. Glynn, we reformulate the count of 10-arcs in terms of the number of $[10_3, n_3]$-superfigurations realizable in that plane: that is, the sets of 10 points and n lines in the projective plane such that each point lies on at least 3 lines, and each line is incident to at least 3 points. We exhibit the 151 unique projective $[10_3, n_3]$-superfigurations, and find conditions on q which determine the number of occurrences of each in $\mathbb{P}^2(\mathbb{F}_q)$. In particular, we show that enumerating the instances of each superfiguration is equivalent to finding the number of distinct solutions in $\mathbb{F}_q^k$ for a system of polynomial equations and inequalities defined over $\mathbb{Z}[x_1, ..., x_k]$, and present methods for counting the roots of these polynomials. These results are of additional interest due to the equivalent formulation of 10-arcs in $\mathbb{P}^2(\mathbb{F}_q)$ as Maximum Distance Separable (MDS) codes of length 10 and dimension 3.

[Joint with Max Weinreich, Susie Kimport]

Specializations of the Lawrence Representations of the Braid Groups at Roots of Unity

Khanh Q Le (kqle@owu.edu)
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Abstract of Report Talk: The braid groups, B_n, were first defined by Emil Artin in 1925 and since then have come to play an important role in many areas of mathematics and physics including topology, geometric group theory, quantum algebras, and conformal field theories. One of the longest open questions related to braid groups was answered in 2000 by Bigelow and Krammer, who proved that the braid groups are linear by exhibiting a faithful representation of B_n on the homology module of a certain configuration space over the ring $\mathbb{Z}[q^{\pm 1}, t^{\pm 1}]$. This representation, known as the Lawrence-Krammer-Bigelow (LKB) representation, is one of a family of homology representations, $H_{n,l}$, first discovered by Ruth Lawrence in 1990. In particular, $H_{n,2}$ is the LKB representation while $H_{n,1}$ is isomorphic to the famous Burau representation $\mathfrak{B}_n$.

In 2011, Jackson and Kerler proved that the Lawrence representations are irreducible over the quotient field $\mathbb{Q}(q, t)$. Moreover, they showed that when the parameters are specialized to $tq = -1$, the Lawrence representations admit a subrepresentation isomorphic to the Temperley-Lieb representation and that the the natural short exact sequence corresponding to this subrepresentation does not split for $n \geq 4$.

In our study we investigate a different specialization of parameters. In particular, we show

that when t is specialized at a primitive root of unity ($t^l = 1$) the Lawrence representations admit a subrepresentation isomorphic to the Burau representation $\mathfrak{B}_n$. Furthermore, we prove that the corresponding short exact sequence

$$0 \to \mathfrak{B}_n \hookrightarrow H_{n,\ell}|_{t^\ell=1} \twoheadrightarrow H_{n,\ell}|_{t^\ell=1}/\mathfrak{B}_n \to 0$$

does not split for $n \geq 4$. However, for $n = 3$ we show that the sequence splits under the condition $q = t$. This proves the existence of a complementary subrepresentation in this case. Of further interest is the relation between this subrepresentation and the subrepresentations previously shown to exist for the specialization $tq = -1$.

A Volume Argument for Tucker's Lemma

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MSRI-UP at UC Berkeley [Mentor: Francis Su]

Abstract of Report Talk: Sperner's lemma is a statement about labeled triangulations of a simplex. McLennan and Tourky (2007) provided a novel proof of Sperner's Lemma using a volume argument and a piecewise linear deformation of a triangulation. We adapt a similar argument to prove Tucker's Lemma on a triangulated cross-polytope P in the 2-dimensional case where vertices of P have different labels. The McLennan-Tourky technique would not directly apply because the natural deformation distorts the volume of P; we remedy this by inscribing P in its dual polytope, triangulating it, and considering how the volumes of deformed simplices behave. We then generalize the argument to apply to triangulated cross-polytopes whose vertices do not have different labels.

[Joint with Christopher Loa]

On a Problem of Sarnak Related to Constructing Efficient Univeral Subsets of 1-Qubit Gates

Qingzhong Liang (qzliang@umich.edu)
University of Michigan [Mentor: Steven Damelin]

Abstract of Report Talk: A 1-qubit gate is defined as an element of the unitary group $U(2)$, the group of complex valued matrices whose conjugate transposes equal their inverses. In quantum computing, it is important to be able to find good approximations of 1-qubit gates by using a dense group generated by a universal subset of the projective special unitary group $G = PSU(2)$. Here, $PSU(2)$ is the group formed by taking all elements of $U(2)$ with determinant 1 and taking the quotient of this subgroup by the group $\{I, -I\}$ where I is the identity matrix. In http://publications.ias.edu/sarnak/paper/2637[1], Sarnak recently studied this problem motivated by developing a continued fraction type algorithm for this problem. The continued fraction algorithm is the fastest known algorithm to approximate real numbers by the dense subset of rational numbers.

In this work, we study the *efficiency* of the approximation of G by using a dense group generated by a universal subset of G. The measure of the efficiency $K(g)$ of an arbitrary universal subset g is defined in [1] and satisfies $K(g) \geqslant 1$. g is more efficient when $K(g)$ is closer to 1.

In this research, we address some open problems posed by Sarnak in [1]. More specifically, we focus on two different universal subsets S and T and estimate their efficiencies as follows:

- We provide many details of the proof of an important conclusion in [1]: $\frac{4}{3} \leqslant K(S) \leqslant 1$.
- In the group generated by T (or S), we explicitly construct the necessary height of

elements that most of $y \in G$ have optimally good approximation. We check it by calculation and programming.

- Also, by theoretical proof, we find that the 1-qubit gate approximation problem is actually related to the problem of computing the covering radius of a certain set of points ν on S^3.
- In order to estimate $K(T)$ and $K(S)$ more precisely, we formulate two conjectures about the covering radius of ν on the 3-sphere. Under the assumption that these conjectures hold, we show the efficiency $K(S)$ and $K(T)$ of the subsets S and T satisfy $K(S) = K(T) = \frac{4}{3}$.

Our preprint containing the details of this work can be found in [arXiv 1506.05785]

Generating Self-Similar Fractals Using the Heisenberg Group

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Cornell University [Mentor: Robert Strichartz]

Abstract of Report Talk: Many familiar fractals are generated by iterated function systems expressed in terms of the standard operations of a Euclidean vector space. This project explores the construction of fractals generated by mappings composed of dilations, rotations, and translations compatible with the Heisenberg group, a non-commutative Lie group acting on $\mathbb{R}^3$. We wrote a MATLAB program to visualize these fractals given "touching conditions," which dictate how the images of the generating functions intersect. Our findings include fractals which project to the Levy Curve and the Twin Dragon Tile onto the xy-plane. We can prove the "Heisenberg Dragon" is finitely ramified, contrasting with the standard Twin Dragon Tile which can only be disconnected with the removal of infinitely many points.

The theory of differential equations on the Twin Dragon Tile does not differ from the classical theory because it is homeomorphic to a disc. Should we find a finitely ramified Heisenberg fractal with cycles, the topological distinctions will allow us to define a new theory of differential equations, mimicking the work of Kigami and Strichartz on the Sierpinski Gasket.

[Joint with Aaron Chen, Weihang Wang]

Modeling Wave Phenomena Using Riccati-System to Construct Nonlinear Schroedinger Equations

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University of Puerto Rico at Mayaguez [Mentor: Erwin Suazo]

Abstract of Poster Presentation: In this poster, similarity transformations are used in order to construct Nonlinear Schrödinger equations admitting bright, dark and Peregrine soliton solutions. We find explicit solutions for a nonlinear Ermakov system with selected variable coefficients to construct the similarity transformations. NLS is the standard model of how light propagates inside of a fiber optic with rich mathematical properties such as being integrable. Some of the applications are pulse dynamics in the dispersion-managed fibers and Bose-Einstein condensates.

[Joint with Gerardo Mercado, Gabriel Amador]

The geometry of discriminants over finite fields

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University of Chicago [Mentor: Benson Farb]

Abstract of Report Talk: In his paper *A Remarkable Geometry of Discriminant Varieties*, Katz concludes that the problem of finding solutions of a polynomial is equivalent to the problem of finding tangent planes to the discriminant hypersurface. In particular, the problem of finding roots to a quadratic is the same as finding tangent lines to the parabola $b^2-4c=0$. Although tangency is difficult to visualize in the same manner over a finite field, the formal derivative allows us to preserve the behavior of a derivative without limits. Therefore the notion of a tangent line and the process to find it remain the same. Hence, the relationship between the problem of finding roots and the problem of finding tangent lines to the discriminant curve translates to any field, in particular the finite field F_q, where $q=p^k$, for some prime p. To depict these results we have created affine planes over F_q and considered all the monic quadratics in the field to answer the question: Is it on the discriminant, does it have two unique solutions or no solution? We have experimented with ways of visualizing the natural structure of F_q. We use Katz' results to bridge the gap between number theory and geometry, particularly to classify arbitrary quadratics in F_q within the above trichotomy.

[Joint with Claudio Gonazles, Marcus Morales]

Quaternionic Ramsey Theory

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Williams College [Mentor: Steven Miller]

Abstract of Report Talk: Classical Ramsey Theory explores constructing as dense as possible sets avoiding a particular interesting property. Famous applications include the Friends and Strangers problem, and numerous questions on coloring graphs. Rankin's 1961 paper formed a bridge between Ramsey Theory and Number Theory by greedily constructing a set of integers avoiding geometric progressions.

These constructions were recently extended by Best et al. to number fields, where consequences of non-unique factorization surface. We extend these ideas further to Hurwitz Quaternions, where the loss of commutativity greatly complicates the arguments and affects the limiting behavior. Though most famous for their applications in physics and geometry, as well as the proof of Lagrange's Four Squares Theorem, the Hurwitz order of quaternions possesses other interesting algebraic properties. This non-commutative ring contains 24 units and an interesting Metacommutation Problem that allows prime factorizations of quaternions that aren't unique in the classical sense of the integers. We construct maximally sized sets of Hurwitz quaternions that avoid geometric progressions up to units and bound their densities, while adjusting for unique properties of the ring. The proofs involve a mix of the algebra of the quaternions with an analysis of the resulting infinite products.

[Joint with Megumi Asada, Eva Fourakis, Eli Goldstein, Nathan McNew]

Constructive Galois Theory with Linear Algebraic Groups

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Eric Y Chen (a5584266@berkeley.edu)
Louisiana State University [Mentor: Morales Jorge]

Abstract of Report Talk: A fundamental aspect of the Inverse Galois Problem is describing all extensions of a base field K with a given Galois group G. A constructive approach to this problem involves the theory of generic polynomials. For a finite group group G, a polynomial $f(t_1,\ldots,t_n;x)\in K(t_1,\ldots,t_n)[x]$ is G-generic if $\mathrm{Gal}(f/K(t_1,\ldots,t_n))\cong G$ and for any Galois

G-extension M/L with $L \supset K$, the parameters $t_1, \ldots, t_n$ can be specialized to L such that f has splitting field M/L. A related notion (due to Saltman) is that of generic extensions, which similarly parametrize all Galois G-extensions containing K. The theory of Frobenius modules, developed by Matzat, is of great practical use for these constructions.

In our work, we use Frobenius modules to show the existence of and explicitly construct generic polynomials and extensions for various groups over fields of positive characteristic. The methods we develop apply to a broad class of connected linear algebraic groups defined over finite fields satisfying certain conditions on cohomology. In particular, we use our techniques to study constructions for unipotent groups, certain algebraic tori and solvable groups, and symplectic groups. An attractive consequence of our work is the construction of generic polynomials in the optimal number of parameters for all cyclic 2-groups over most fields of positive characteristic. This contrasts with a theorem of Lenstra, which states that no cyclic 2-group of order ≥ 8 has a generic polynomial over $\mathbb{Q}$.

[Joint with J.T. Ferrara]

Modeling wave phenomena using non-autonomous nonlinear Schroedinger equations

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UPR Mayaguez Campus [Mentor: Erwin Suazo]

Abstract of Poster Presentation: In this poster we describe the evolution of waves by constructing non-autonomous nonlinear Schroedinger equations (NLS) with variable coefficients. To construct these equations we will be using a Riccati system of equations (nonlinear as well) with selected variable coefficients with explicit solutions and use of generalized lens transformations to turn them into explicit solutions for the NLS. Ultimately the NLS will be of standard form i.e with constant coefficients. These transformations will also create parameters for the different NLS equations. These will of course be explained in detail as they play an important part in the dynamics of the moving wave. Aside from the analytical solution to the NLS, we will also use numerical approximation and dynamical analysis to describe the wave phenomena in both a quantitative and qualitative way. These equations will also provide practical applications in many areas. Such as:

Bose-Einstein condensates
Dispersion-managed optical fibers and soliton lasers
Pulse dynamics in the dispersion-managed fibers

This research, done at UPRM, is an MAA activity funded by NSF (grant DMS-1359016) and NSA (grant DMS-1359016).

[Joint with Kiara Colon, Nathalie Luna]

A Ramsey Theoretic Approach to Function Fields Structure

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Williams College [Mentor: Steven Miller]

Abstract of Poster Presentation: Ramsey theory concerns itself with how large a set needs to be for a certain structure to arise. The classic Friends and Strangers problem asks for how many people need to be at a party to guarantee 3 mutual friends or 3 mutual strangers. Ramsey theory can also be applied to understanding the structure of algebraic rings. Rankin used a greedy method to construct a large set that avoids 3-term geometric progressions

b, rb, r^2b in the integers. This was generalized just last year to integer rings of number fields. In our work, we resolve analogous problems for polynomials over finite fields.

We build off prior work to construct, for any $\mathbb{F}_q[x]$, a large greedy set free of 3-term progressions. We also give upper and lower bounds on the supremum of upper densities of 3-term progression-free sets. New features emerge in this setting – the proofs are distinctly combinatorial, a feature not seen in other cases. We take advantage of the combinatorics arising from finite characteristic through counting q-smooth elements and the number of irreducible polynomials in $\mathbb{F}_q[x]$, leading to cleaner, more illuminating formulas and constructions. Namely, while in other settings the bounds need to be calculated individually or in cases, our formulas are in terms of general q. As a result, we prove how the supremum changes as $q = p^n$ varies in size and as $n \to \infty$.

[Joint with Eva Fourakis, Eli Goldstein, Sarah Manski, Nathan McNew]

Escape Rates and Equilibrium States for Open Topological Markov Chains

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Fairfield University [Mentor: Mark Demers]

Abstract of Report Talk: Open dynamical systems are models of physical systems in which mass or energy is allowed to escape from the system. Central questions involve the existence of conditional equilibria (measures that are invariant under the dynamics conditioned on non-escape) which can be realized as limiting distributions under the dynamics of the open system. We study this problem in the context of topological Markov chains, which are a class of symbolic dynamical systems with a wide variety of applications. Under a combinatorial condition on the Markov chain and for positive recurrent potentials, we prove the existence (and uniqueness in a certain class) of limiting distributions which represent conditional equilibria for the open system. We also prove a relation between the escape rate from the system and the entropy on the survivor set (the set of points that never enters the hole).

[Joint with Elizabeth Yoo, Phil Mayer]

Zarankiewicz Numbers and Bipartite Ramsey Numbers

Alexander W Neal Riasanovsky (alexneal@math.upenn.edu)
Rochester Institute of Technology [Mentor: Stanislaw Radziszowski]

Abstract of Report Talk: The Zarankiewicz number $z(b; s)$ is the maximum size of a subgraph of $K_{b,b}$ which does not contain $K_{s,s}$ as a subgraph. The two-color bipartite Ramsey number $b(s, t)$ is the smallest integer b such that any coloring of the edges of $K_{b,b}$ with two colors contains a $K_{s,s}$ in the first color or a $K_{t,t}$ in the second color.

In this work, we design and exploit a computational method for bounding and computing Zarankiewicz numbers. Using it, we obtain several new values and bounds on $z(b; s)$ for $3 \le s \le 5$. Our approach and more knowledge about $z(b; s)$ permit us to improve some of the results on bipartite Ramsey numbers obtained by Goddard, Henning and Oellermann in 2000. In particular, we compute the smallest previously unknown bipartite Ramsey number, $b(2, 5) = 17$. Moreover, we prove that up to isomorphism there exists a unique 2-coloring which witnesses the lower bound $16 < b(2, 5)$. We also find the tight bounds $17 \le b(2, 2, 3) \le 18$, the smallest open case for multicolor bipartite Ramsey numbers.

[Joint with Alex F. Collins, John C. Wallace]

Computing the Evans function via solving a linear boundary value ODE

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Brown University [Mentor: Bjorn Sandstede]

Abstract of Report Talk: Traveling waves occur in many natural phenomena; determining their stability in partial differential equation models is an important and interesting problem. Stability can be analyzed by linearizing the underlying model about the wave: if the resulting linear operator does not have eigenvalues with positive real part, the wave will be stable. We are interested in developing algorithms to test stability numerically. For waves in one spatial dimension, one may form an Evans function - an analytic Wronskian-like function whose zeros correspond to the eigenvalues of the linearized system and show that its winding number is zero when computed along appropriate contours. Currently, two methods exist for calculating the Evans function numerically, namely the exterior-product method (which is very expensive numerically) and the method of continuous orthogonalization (in which a nonlinear rather than the original linear system is solved). We develop two new algorithms for computing the Evans function through appropriate linear boundary-value problems that are cheaper than the previous methods and prove that they preserve analyticity of the Evans function. We also implemented these algorithms and tested them on classical systems.

[Joint with Nathaniel Ventura, Blake Barker]

Decompositions of Parking Functions

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UC Santa Barbara [Mentor: Michael Dougherty]

Abstract of Poster Presentation: There is a well-known bijection between parking functions of size n (denoted PF_n) and the maximal chains of the noncrossing partition lattice of height $n+1$ (denoted NC_{n+1}). This lattice has interesting properties such as self-duality. Using this bijection we explore a particular decomposition of PF_n and the corresponding maximal chains in NC_{n+1}. We show that the posets formed by these maximal chains preserve interesting properties of NC_{n+1}. We also provide several formulas that enumerate these posets and decompositions, making use of the Catalan numbers. In addition, we provide interpretations of this particular decomposition in objects such as labeled Dyck paths, labeled rooted forests, and nonnesting partitions.

[Joint with Ryo Kudo, Melody Bruce]

Analysis of Interfaces for the Nonlinear Double-Degenerate Reaction-Diffusion Equation

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Florida Institute of Technology [Mentor: Ugur Abdulla]

Abstract of Poster Presentation: We consider the problem of interface development and local asymptotics of solutions near the interface in the following Cauchy problem for the nonlinear double-degenerate parabolic PDE with reaction term:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\Big(\Big|\frac{\partial u^m}{\partial x}\Big|^{p-1}\frac{\partial u^m}{\partial x}\Big) - bu^\beta, x \in \mathbb{R}, t > 0; u|_{t=0} = C(-x)_+^\alpha \tag{3}$$

The problem arises in many applications in physics, biology, and chemistry; examples include heat radiation in plasma, spatial spread of populations, diffusion of chemicals through

groundwater, etc. The structure of the PDE implies that the interface behavior is determined by the competition between two opposing factors: nonlinear diffusion and absorption or reaction. The full solution of this problem for the reaction-diffusion equation ($p = 1$ in (1) was given in 2000 [Abdulla and King, SIAM J. Math. Anal., 32, 2(2000), 235-260] and 2002 [Abdulla, Nonlinear Analysis, 50, 4(2002), 541-560]. The goal of this project is to apply the methods of these papers to solve the open problem for the double-degenerate reaction-diffusion equation ($p > 1, mp > 1$). First we apply the nonlinear scaling method to identify the outcome of the competition between the two opposing factors in various regions of the (α, β)-parameter space. We prove that for $\alpha < \frac{1+p}{mp-\min\{1,\beta\}}$, diffusion dominates and the interface expands, while for $\alpha > \frac{1+p}{mp-\beta}, 0 < \beta < 1$, reaction dominates and the interface contracts. On the borderline case, when $\alpha = \frac{1+p}{mp-\beta}, b > 0, 0 < \beta < 1$, the interface may initially expand, shrink, or remain stationary, depending on the parameter C. It is proved that there is a critical $C*$ determined by the parameters of the problem such that for $C < C*$ and $C > C*$, the interface shrinks and expands respectively, while for $C = C*$, the solution is stationary. Moreover, in this borderline case, when $p(m + \beta) = 1 + p$, we derive an explicit global traveling wave solution. In all cases, we prove explicit formulae for the interface and local solution, with precise estimations up to constant coefficients. The rigorous proof methods that we use are rescaling, construction of super- and subsolutions and special comparison theorems in irregular domains.

[Joint with Adam Prinkey]

Time-Inhomogeneous Branching Processes

Mark Perlman (markperlman@uchicago.edu)
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Abstract of Poster Presentation: A branching process Z_n is a probability model for a population defined by $Z_0 = 1$ and $Z_{n+1} = \sum_{j=1}^{Z_n} X_{n,j}$ where $X_{n,j}$ are i.i.d. copies of X_n, the offspring distribution at time n. In the classical time-homogeneous theory, where $X_n = X$ is independent of n, two results hold regarding the long-term behavior of the process: first, extinction (Z_n is eventually 0) occurs with probability one if and only if $E(X) \leq 1$; second, a normalized process conditioned on survival, $\frac{Z_n|Z_n\neq 0}{E(Z_n|Z_n\neq 0)}$, converges to an exponential distribution if $E(X) = 1$. We extend these results to the time-inhomogeneous case for both discrete- and continuous-time branching processes. The specific case when the generating function is fractional linear lends itself to explicit calculations. Then, general results follow from bounding by fractional linear processes. We find that extinction is sure if and only if $E(Z_n)^{-1}$ is not summable; and the scaled, conditioned process converges to an exponential if both $\sum_{i=1}^{n} E(Z_i)^{-1}$ and $E(Z_n)\sum_{i=1}^{n} E(Z_i)^{-1}$ tend to infinity.

[Joint with Prof. Dmitry Dolgopyat, Nick Battacharya and Monica Gorman]

Frechet Differentiability in Optimal Control of Parabolic Free Boundary Problems

Jessica A Pillow (pilja-16@rhodes.edu)
Florida Institute of Technology [Mentor: Ugur Abdulla]

Abstract of Report Talk: We consider an optimal control of the Stefan type free boundary problem for the following general second order linear parabolic PDE:

$$\frac{\partial}{\partial x}\Big(a(x,t)\frac{\partial u}{\partial x}\Big) + b(x,t)\frac{\partial u}{\partial x} + c(x,t)u - \frac{\partial u}{\partial t} = f(x,t)$$

where $u(x,t)$ is the temperature function. The density of heat sources f, unknown free boundary, and boundary heat flux are components of the control vector, and the cost functional consists of the L_2-declination of the trace of the temperature at the final moment, the

temperature at the fixed boundary and the final position of the free boundary from available measurements. This problem arises when considering a phase transition process with unknown temperature function, phase transition boundary, source term and boundary heat flux. We follow a new variational formulation developed in *U. G. Abdulla, Inverse Problems and Imaging, 7,2(2013),307-340.*

In this project we prove Frechet differentiability of the cost functional in Hilbert space framework. Extension of the differentiable calculus to the infinite-dimensional setting is the major mathematical challenge in this context, especially due to the fact that free boundary is the component of the control vector, and the increment of the cost funtional must take into account the variation of the domain of dependence. We apply the idea of decomposition of the domain, possibly into countably many subdomains depending on the sign oscillations of the free boundary increment, and we analyze carefully the effect of the boundary integrals on the derivation of the first variation of the cost functional. We introduce the notion of adjoint problem, as an infinite dimensional analogy of the classical Lagrange multipliers in finite dimensional constrained optimization problems. With the delicate use of sharp embedding theorems in fractional Sobolev-Besov spaces we prove the Frechet differentiability of the cost functional, and derive the formula for the Frechet gradient expressed in terms of the traces of the state vector and the solution of the adjoint problem. Our Frechet differentiability result can be used for implementation of the projective gradient method in Hilbert spaces setting for the numerical solution of the problem.

[Joint with Dylanger Pittman, Jim Jones, Jonathan Goldfarb]

New properties of branched covers of surfaces

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Abstract of Report Talk: A landmark result of Hurwitz from 1891 shows that there are only finitely many topological equivalence classes of branched covers $X \to Y$ of any prescribed degree n between closed connected orientable surfaces of prescribed genera. For fixed values of n and the genus of X, representatives for the equivalence classes are given by the *branching type* of the cover, which is a collection of partitions of n satisfying certain properties. Many authors have studied the problem of describing which collections of partitions arise as the branching type of a cover, which is equivalent to understanding the topological nature of all branched covers. We introduce new techniques which yield new constraints on the branching types. In particular, we exhibit the first infinite sequence of batches of partitions which do not occur as a branching type but which do satisfy the necessary conditions due to Hurwitz for being the branching type of a cover $X \to Y$ with X of genus at least 2. Our arguments use the representation theory of S_n, results about primitivity and multiple transitivity of permutation groups, and a study of batches of conjugacy classes in S_n.

[Joint with Yifan Wu]

Shifting Numerical Semigroups and the Catenary Degree

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Abstract of Report Talk: Numerical monoids (co-finite additive submonoids of $\mathbb{N}$) are known to have non-unique factorizations into irreducible elements. To quantify this interesting factorization theory, the catenary degree measures how many irreducible elements different factorizations for the same element are not shared in common. For a numerical monoid S, the catenary degree of S, denoted $\mathsf{c}(S)$, is the largest catenary degree of any element. In this

talk, we investigate numerical monoids of the form $M_n = \langle n, n+r_1, n+r_2, \ldots, n+r_k \rangle$ with shift parameter n. We show that the sequence of catenary degrees $\{\mathsf{c}(M_n)\}_{n\in\mathbb{N}}$ for M_n has an eventual quasi-linear behavior. In the process of studying this catenary degree sequence, we prove a broader result of interest in commutative algebra: the Betti elements and minimal presentations of these shifted numerical monoids is eventually periodic with period r_k.

[Joint with Rebecca Conaway, Felix Gotti]

Rankings of Cartesian Product $(K_s - e)$ x P_n

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Abstract of Poster Presentation: A ranking of a graph is a vertex labeling such that every path between vertices with the same label contains a vertex with a larger label. The rank number of a graph is the smallest possible number of labels in a ranking. Rankings were first introduced in 1995 by Bodlaender et al. for the purpose of studying algorithms. For a given graph G and a positive integer t, the question whether the rank number is less than or equal to t is NP-complete for many classes of graphs. Interest in rankings of graphs was sparked by their many applications to other fields including designs of very large scale integration layouts (VLSI), Cholesky factorizations of matrices in parallel, and scheduling problems of assembly steps in manufacturing systems. While rank numbers for some families of graphs are established, there exist numerous families of graphs, for which the rank numbers remain unknown. Lately, families of Cartesian products are being investigated for their rank numbers.

In 2010 Alpert established rank numbers for the Cartesian product of complete graphs and paths. We studied the rankings of $(K_s - e) \times P_n$. In a ranking of $(K_s - e) \times P_n$, the presence of non-adjacent vertices in $K_s - e$ allows repeated labels in any $(K_s - e)$-layer. For example, $(K_s - e) \times P_1$ uses $s - 1$ labels, where the non-adjacent vertices share the same label. However, as n increases, this pattern cannot be extended, and that makes finding the rank numbers of $(K_s - e) \times P_n$ significantly challenging. We established the rank numbers of $(K_s - e) \times P_n$ for all values of s and n using the concepts of connectible ends among others.

Zeros of the Modular Form $E_k E_l - E_{k+l}$

Sarah C Reitzes (sreitzes@verizon.net)
Texas A and M University [Mentor: Matthew Young]

Abstract of Report Talk: The Eisenstein series of weight k is a modular form defined by $E_k(z) = \frac{1}{2}\sum\sum(cz+d)^{-k}$, where the sum is taken over all relatively prime integers c and d. Rankin and Swinnerton-Dyer proved that all of the zeros of the Eisenstein series in the fundamental domain (the intersection of the strip $-\frac{1}{2} \le x \le \frac{1}{2}$ with the region $|z| \ge 1$) lie on the arc $|z| = 1$. In this study we consider the zeros of the weight $k + l$ modular form $E_k E_l - E_{k+l}$ in the fundamental domain. Based on numerical evidence, we conjecture that all of its zeros lie on the boundary. In the extreme case where $l = 4$, we conjecture that all of the zeros lie on the arc $|z| = 1$, while at the opposite extreme where $k = l$, we conjecture that all of the zeros lie on the lines $x = \pm\frac{1}{2}$. Using asymptotoic approximations, we have proved that for $l = 4, 6$, and 8 all of the zeros lie on the arc $|z| = 1$ and that for $k = l$, almost all of the zeros lie on the lines $x = \pm\frac{1}{2}$.

[Joint with Polina Vulakh, Bard College]

The Combinatorics of Borel Sets in a Fixed Degree

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Mount Holyoke College [Mentor: Jessica Sidman]

Abstract of Report Talk: An ideal is an algebraic object that is closed under addition and multiplication in a ring. We focus on Borel-fixed ideals, a subset of monomial ideals whose behavior satisfies nice combinatorial properties.

If for any monomial m in n variables $x_1 > x_2 > ... > x_n$ in a set S, whenever $x_j|m$ and $x_i > x_j$, $m\frac{x_i}{x_j}$ is also in S, then we say that S satisfies the Borel property. We refer to S as a Borel-fixed set. If the generators of an ideal I form a Borel-fixed set, then I is called a Borel-fixed ideal. Expanding our knowledge of Borel-fixed ideals and the Borel-fixed sets that generate them may translate to a better understanding of this important class of ideals.

Our research investigates the structure of Borel-fixed sets in terms of partial ordering. We demonstrate that the partially ordered set of all monomials of degree d in n variables is a lattice, and is graded with rank function $r(m = x_1^{a_1}x_2^{a_2}...x_n^{a_n}) = \sum_{i=1}^{n}(i-1)a_i$. Additionally, we consider methods for counting the number of Borel-fixed sets of degree d in n variables, and show that there are $d+1$ Borel-fixed sets of degree d in two variables and $2^{d+1}-1$ Borel-fixed sets of degree d in three variables.

[Joint with Maya Urbschat]

Differentially Private Graphical Model Selection

Zexi Song (zs0811@bard.edu)
Rutgers University [Mentor: Anand Sarwate]

Abstract of Poster Presentation: Given a discrete dataset with n individuals taking values on m different features, we want to construct a graphical model that captures the dependencies and correlations among the features at the population level while at the same time preserving the privacy of individuals in the data. The privacy model we use is differential privacy. We develop a differentially private version of the Chow-Liu algorithm which finds the best tree-structured graphical model fitting the data. It works by computing pairwise mutual informations between the feature variables, yielding a weighted complete graph on m vertices, one for each feature. It then selects the maximum weight spanning tree to preserve the maximum correlations in the data. To make this differentially private we add Laplace noise to the computed mutual informations. In order to determine the parameter for the Laplace distribution to yield ϵ-differential privacy we compute a tight upper bound on the sensitivity of the plug-in estimate of the mutual information function. In particular, we prove that in our setting the sensitivity of mutual information between random variables X, Y is bounded by,

$$\Delta_n(I(X;Y)) \leq 2\left(\frac{1+1/n}{2}\log(\frac{1+1/n}{2}) - \frac{1-1/n}{2}\log(\frac{1-1/n}{2}) - \frac{1}{n}\log(\frac{1}{n})\right)$$

where n is the sample size. This shows that the sensitivity does not depend on the alphabet size and drops to 0 very fast as n increases. For moderate privacy risk ϵ our simulations with Python show that we can accurately recover the graphical structure for a planted-tree model. We also evaluate the approach on the Adult dataset from the UCI Machine Learning Repository. The majority of the tree structure we recovered is rather robust even when the privacy level is high.

Locating Numbers and Sets for Disconnected Graphs

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Abstract of Poster Presentation: The locating code for a vertex v in a finite graph $G(V,E)$ is a vector representing distances of v with respect to vertices in an ordered subset W of $V(G)$. We call W a locating set if each vertex in the graph has a distinct locating code and we call the number of vertices in the minimal locating set the locating number. The concepts of locating sets and numbers were developed by Pirzada et. al. in order to study graphical representations of commutative rings. Previous work involving locating sets has depended on the assumption that $G(V,E)$ is connected. We prove conditions for locating sets of disconnected graphs and present a closed form for computing the locating number of any undirected graph. Furthermore, we give characterization theorems for graphs of particular locating numbers, which generalize past results by Pirzada et. al. Finally, we present an algorithm we have developed in Sage for computing locating numbers and sets for a graph.

Nonexistence results for branched covers

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University of Michigan [Mentor: Michael Zieve]

Abstract of Report Talk: Surfaces play a central role in various fields of mathematics. Many questions in algebraic geometry, complex analysis, low-dimensional topology, and field theory involve maps between surfaces called *branched covers*. These maps are locally n-to-1 except at a finite set of points in their target space, called the *branch points*. Hurwitz established a correspondence between equivalence classes of branched covers and finite collections of partitions of n describing the behavior of a branched cover over its branch points, and a problem which has remained open for over 100 years is the determination of which collections of partitions can occur for a branched cover. The easy necessary conditions for a branching type to occur (called the Riemann–Hurwitz conditions) are believed to usually be sufficient, so it is of primary interest to determine the relatively few plausible branching types which do not occur.

We exhibit eight large non-occurring infinite families of plausible branching types. These results comprise far-reaching generalizations of previous non-existence results by Pascali–Petronio, Pervova–Petronio, Pakovich, and Zheng, and special cases of our results resolve several conjectures from the literature. Our results give hope for a complete description of all non-occurring plausible branching types, since they explain the vast majority of all low-degree examples of non-occurring plausible branching types which are not already ruled out by decomposability constraints. Our proofs use various methods, ranging from explicit arguments with permutations to the use of Grothendieck's theory of *dessins d'enfants*, which establishes a correspondence between branched covers from a genus-g surface to the sphere and graphs embedded on a genus-g surface.

Dispersal-induced global extinction in two-patch model with Allee effect

Margaret E Swift (meswift@email.wm.edu)
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Abstract of Poster Presentation: Centuries of overfishing and the gradual destruction of habitat have led to the rapid devastation of the Chesapeake Bays oyster supply. We study asymmetric dispersal between two coupled patches (oyster reefs) under the Allee effect. This effect is displayed by many species, and is one under which initial populations below a certain

threshold decline, while those above can persist. We extend a previous ordinary differential equation model with symmetric dispersal rates between patches (Kang and Lanchier, Bull. Math. Biol. 2011), and explore the steady state bifurcation structure while varying the dispersal rates and Allee threshold. We also show analytically that there are no periodic orbits. At high Allee thresholds, we find large parameter ranges in which the extinction state is the only fixed point. Previous symmetric models did not uncover this behavior, and it raises concerns for environmental restoration of other species that may exhibit the Allee effect and asymmetric dispersal, such as in estuarine and marine systems.

[Joint with Junping Shi, Rom Lipcius]

Benford's Law in Fragmentation Processes and Other Systems

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Williams College [Mentor: Steven Miller]

Abstract of Poster Presentation: Since the late 1800s it has been known that the first digits of many sets of data are not distributed uniformly, as might be expected, but follow a logarithmic bias. This phenomenon is now known as Benford's law. Those sets to which Benford's law applies include both physical and number theoretic data. No universal theory of Benford behavior is understood; many explanations for the emergence of Benford behavior have been proposed, and different sets seem to exhibit Benford behavior for different reasons.

In 1985, Lemons suggested that Benford's law should arise out of the decompositions of conserved quantities. In 2013, Becker et al. helped develop this idea in a rigorous way by showing that the continuous fragmentation of a length results in pieces whose lengths are Benford-distributed. We extend their one-dimensional results and exploit a unity between the one dimensional and higher dimensional case to obtain results about more complex continuous fragmentation processes. In particular, the systems we study capture many features of physical decomposition processes. The difficulty is in managing the significantly more complicated network of dependencies that arises in the higher-dimensional case, which is accomplished by choosing fragmentation points in such a way that we can appeal to convexity results. Additionally, we resolve Becker et al.'s conjecture regarding discrete fragmentation processes.

In addition, we study the relationship between Benfordness and natural density. Cohen showed that the integers are Benford under all reasonable densities. We extend this result to include all subsets on the natural numbers with positive natural density. In 1973 Serre communicated a result, due to Bombieri, that the primes are Benford in analytic density. We embed this result in a more general study of zero-density subsets of the natural numbers. Finally, we carry out an analogous study of subsets of the reals.

[Joint with Eyvindur Palsson]

Low-Lying Zeros of Families of L-Functions

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Williams College [Mentor: Steven Miller]

Abstract of Report Talk: The Katz and Sarnak philosophy suggests that the limiting behavior of zeros of families of L-functions are well modeled by the eigenvalues of random matrix ensembles drawn from the classical compact groups; these zeros play a key role in problems from counting primes to understanding the class number. In 2006, Miller discovered a surprising discrepancy between the lowest-lying zeros of elliptic curve L-functions and the lowest-lying eigenvalues of the corresponding random matrix ensemble at finite conductor. Dueñez et al. pushed the correspondence further by showing that we can model also the lower-order terms

of the one-level density of elliptic curve L-functions by the lower-order terms in the one-level density of a matrix ensemble. To capture the behavior of the first lower-order term, Dueñez et al. needed only to rescale the size of their random matrices; to correctly model the further lower-order terms they needed to introduce a new parameter to reflect the discretization of central values.

We extend their work and investigate the family of Dirichlet L-functions associated to quadratic characters. Rubinstein's thesis shows a slight discrepancy between the lowest-lying zeros of these L-functions and the lowest-lying eigenvalues of the symplectic matrix ensemble. Following Dueñez et al., we hoped to resolve this discrepancy by rescaling the size of the matrices relative to the conductor of the L-functions. Surprisingly, our preliminary calculations suggest that the first lower-order terms of the one-level densities of the L-functions and the random matrices are opposite in sign, and therefore that our desired matrix size is negative. This obstruction, coming as it does in the first lower-order term, is more serious than that encountered in the elliptic curve families. We therefore explore the possibility of developing a new, modified matrix ensemble that models correctly the behavior of the lower-order terms.

[Joint with Owen Barrett, Kevin Yang]

Lazy Cops and Robbers on Chess and Product Graphs

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Abstract of Report Talk: The pursuit-evasion game of Cops and Robbers on graphs has been studied extensively since its introduction in the 1980s. A recently-proposed variant, "Lazy Cops and Robbers", allows just one cop to move per round. We studied several classes of graphs, looking for possible differences between the Ordinary and Lazy versions; in both scenarios we seek the *cop number* (denoted c or c_L, respectively), the minimum number of cops needed to guarantee victory. We have proven sharp upper bounds for c_L of graphs obtained by Cartesian and strong products, and we have established algorithmic results for specific graphs, e.g. $c_L(C_n \Box P_m) = 2$. We also considered chess graphs and found a relationship between c_L and their domination numbers. Furthermore, we have obtained a characterization of c for Queen's graphs, proving that $\forall n \geq 7$. $3 \leq c(Q_n) \leq 4$ and $\forall n \geq 19$. $c(Q_n) = 4$. Finally, we propose conjectures and open problems related to planar graphs, n-fold products, and domination numbers.

Rational Functions with Many Critical Values

Karl G Winsor (krlwnsr@umich.edu)
University of Michigan [Mentor: Michael Zieve]

Abstract of Poster Presentation: A guiding principle in one-dimensional complex dynamics is that the behavior of a rational function under iteration is determined to a large extent by the behavior of the critical points. This leads to the question of which configurations of critical points are possible. More generally, for any degree-n rational function $f(x)$ and any complex number c, we write $E_f(c)$ for the multiset of multiplicities of the zeroes of $f(x) - c$. A fundamental problem is to determine all possibilities for the collection of pairs $(c, E_f(c))$ with $c \in \mathbb{C} \cup \{\infty\}$, as f varies over all nonconstant rational functions. We solve this problem for degree-n rational functions which have at least $n/2 + 2$ critical values. The conclusion is that the well-known necessary conditions for a collection to occur are actually sufficient in this situation. We also show that the conclusion would be different for functions having $n/2 + 1$ critical values, so that our result is best possible. Our result resolves a conjecture of Zheng when n is even, and improves a weaker result of Baranski. Our proof involves a novel combination of ideas of Hurwitz, Edmonds–Kulkarni–Stong, and Boccara in order to reduce

the problem to the question of determining when a tuple of conjugacy classes in S_n contains elements whose product is a cycle, which we resolve via new techniques.

Meromorphic maps between Riemann surfaces having a cyclic branch cycle

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Abstract of Report Talk: Starting with Riemann's work, there has been great interest in meromorphic maps between compact Riemann surfaces. Associated to any such map of degree n is a batch of *branch cycles*, which are elements of S_n satisfying certain properties. We exhibit both necessary conditions and sufficient conditions for a batch of elements of S_n to occur as the branch cycles of such a meromorphic map. For instance, we determine all 4-tuples of elements of S_n which occur as such branch cycles and which include a k-cycle for some $k \geq n/2$. A special case of this result presents infinitely many tuples of elements of S_n which do not occur as branch cycles even though they satisfy all the known necessary conditions for occurring in this manner. Thus, our result shows that these nonoccurring tuples are isolated from any other nonoccurring tuples. This result improves a result of Boccara. We also give results showing how to produce occurring $(k+1)$-tuples from occuring k-tuples, which generalizes results of Baranski.