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## FRECHET DIFFERENTIABILITY IN OPTIMAL CONTROL OF PARABOLIC FREE BOUNDARY PROBLEMS

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**Abstract of Report Talk:** We consider an optimal control of the Stefan type free boundary problem for the following general second order linear parabolic PDE:

$$\frac{\partial}{\partial x} \left( a(x, t) \frac{\partial u}{\partial x} \right) + b(x, t) \frac{\partial u}{\partial x} + c(x, t)u - \frac{\partial u}{\partial t} = f(x, t)$$

where  $u(x, t)$  is the temperature function. The density of heat sources  $f$ , unknown free boundary, and boundary heat flux are components of the control vector, and the cost functional consists of the  $L_2$ -declination of the trace of the temperature at the final moment, the temperature at the fixed boundary and the final position of the free boundary from available measurements. This problem arises when considering a phase transition process with unknown temperature function, phase transition boundary, source term and boundary heat flux. We follow a new variational formulation developed in *U. G. Abdulla, Inverse Problems and Imaging, 7,2(2013),307-340*.

In this project we prove Frechet differentiability of the cost functional in Hilbert space framework. Extension of the differentiable calculus to the infinite-dimensional setting is the major mathematical challenge in this context, especially due to the fact that free boundary is the component of the control vector, and the increment of the cost functional must take into account the variation of the domain of dependence. We apply the idea of decomposition of the domain, possibly into countably many subdomains depending on the sign oscillations of the free boundary increment, and we analyze carefully the effect of the boundary integrals on the derivation of the first variation of the cost functional. We introduce the notion of adjoint problem, as an infinite dimensional analogy of the classical Lagrange multipliers in finite dimensional constrained optimization problems. With the delicate use of sharp embedding theorems in fractional Sobolev-Besov spaces we prove the Frechet differentiability of the cost functional, and derive the formula for the Frechet gradient expressed in terms of the traces of the state vector and the solution of the adjoint problem. Our Frechet differentiability result can be used for implementation of the projective gradient method in Hilbert spaces setting for the numerical solution of the problem.

[Joint work with Dylanger Pittman, Jim Jones and Jonathan Goldfarb]

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