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Analysis of Interfaces for the Nonlinear Double-Degenerate Reaction-Diffusion Equation

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Abstract of Poster Presentation: We consider the problem of interface development and local asymptotics of solutions near the interface in the following Cauchy problem for the nonlinear double-degenerate parabolic PDE with reaction term:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\Big(\Big|\frac{\partial u^m}{\partial x}\Big|^{p-1}\frac{\partial u^m}{\partial x}\Big) - bu^\beta, x \in \mathbb{R}, t > 0; u|_{t=0} = C(-x)_+^\alpha \tag{1}$$

The problem arises in many applications in physics, biology, and chemistry; examples include heat radiation in plasma, spatial spread of populations, diffusion of chemicals through groundwater, etc. The structure of the PDE implies that the interface behavior is determined by the competition between two opposing factors: nonlinear diffusion and absorption or reaction. The full solution of this problem for the reaction-diffusion equation ($p = 1$ in (1) was given in 2000 [Abdulla and King, SIAM J. Math. Anal., 32, 2(2000), 235-260] and 2002 [Abdulla, Nonlinear Analysis, 50, 4(2002), 541-560]. The goal of this project is to apply the methods of these papers to solve the open problem for the double-degenerate reaction-diffusion equation ($p > 1, mp > 1$). First we apply the nonlinear scaling method to identify the outcome of the competition between the two opposing factors in various regions of the (α, β)-parameter space. We prove that for $\alpha < \frac{1+p}{mp-\min\{1,\beta\}}$, diffusion dominates and the interface expands, while for $\alpha > \frac{1+p}{mp-\beta}, 0 < \beta < 1$, reaction dominates and the interface contracts. On the borderline case, when $\alpha = \frac{1+p}{mp-\beta}, b > 0, 0 < \beta < 1$, the interface may initially expand, shrink, or remain stationary, depending on the parameter C. It is proved that there is a critical $C*$ determined by the parameters of the problem such that for $C < C*$ and $C > C*$, the interface shrinks and expands respectively, while for $C = C*$, the solution is stationary. Moreover, in this borderline case, when $p(m+\beta) = 1+p$, we derive an explicit global traveling wave solution. In all cases, we prove explicit formulae for the interface and local solution, with precise estimations up to constant coefficients. The rigorous proof methods that we use are rescaling, construction of super- and subsolutions and special comparison theorems in irregular domains.

[Joint work with Adam Prinkey]

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