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Evolution of Free Boundaries for the Nonlinear Fokker-Planck Equation

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Abstract of Report Talk: We investigate the problem on short-time behavior of free boundaries and local solutions near it in the following Cauchy problem for the nonlinear degenerate diffusion equation with convection:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u^m}{\partial x^2} + b\frac{\partial u^\gamma}{\partial x}, x \in \mathbb{R}, t > 0; u|_{t=0} = C(-x)_+^\alpha \tag{1}$$

where $m > 1, \gamma > 0$. The PDE in (1) is called the nonlinear Fokker-Planck equation. It arises in many applications in physics, biology, and chemistry; examples include infiltration of water in a homogeneous porous medium, transport of thermal energy in plasma, spatial transport of populations, etc. The problem of determining the short-time behavior of the free boundaries, or interfaces, is known as the Barenblatt problem. Full solution of this problem for the reaction-diffusion equation was given in [Abdulla and King, SIAM J. Math. Anal., 32, 2(2000), 235-260] and [Abdulla, Nonlinear Analysis, 50, 4(2002), 541-560], but the problem is open for reaction-diffusion-convection equations. The goal of this project is to apply the methods of these papers to solve the open problem for the diffusion-convection equation (1). It is proved that the behavior of the interface in the case $b > 0$ depends on the outcome of the competition between two opposing factors: diffusion and convection. By applying the scaling method we identify various regions of the (α, γ)-parameter space where one of the factors dominate over the other. It is proved that if either $0 < \gamma < \frac{m+1}{2}, 0 < \alpha < \frac{1}{m-\gamma}$ or $\gamma \geq \frac{m+1}{2}, 0 < \alpha < \frac{2}{m-1}$, then diffusion dominates and the interface expands. If $\alpha > \frac{1}{m-\gamma}$ then convection dominates and the interface either shrinks or remains stationary. On the borderline case, when $\alpha = \frac{1}{m-\gamma}, 0 < \gamma < \frac{m+1}{2}$, the interface may initially expand, shrink or remain stationary, depending on the parameter C. It is proved that there is a critical $C*$ determined by the parameters of the problem such that for $C < C*$ and $C > C*$, the interface shrinks and expands respectively, while for $C = C*$, the solution is stationary. In all cases, we prove explicit formulae for the interface and local solution, with precise estimations up to constant coefficients. The rigorous proof methods that we use are rescaling, construction of super- and subsolutions and special comparison theorems in irregular domains. A WENO scheme was applied to the problem and the numerical results support our proved estimations.

[Joint work with Adam Prinkey, Jonathan Goldfarb]

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