

YOUNG MATHEMATICIANS CONFERENCE 2015

The Ohio State University, August 21-23

ON THE FINE CLASSIFICATION OF PERIODIC ORBITS OF CONTINUOUS ENDOMORPHISMS AND UNIVERSALITY IN CHAOS

Muhammad Abdulla (abdullar@sas.upenn.edu)
Alyssa L Turnquist (alt007@morningside.edu)
Florida Institute of Technology [Mentor:Ugur Abdulla]

Abstract of Poster Presentation: We complete the classification of the periodic orbits of period $2^n(2k + 1)$, $k > 1$, of the continuous endomorphisms on the real line which are minimal with respect to Sharkovski ordering. By developing the new constructive method suggested recently in *Abdulla et al. J. of Diff. Equat. and Appl., 19,8(2013), 1395-1416*, it is proved that independent of k , there are $2^{2^{n+1}-2}$ types of digraphs (and cyclic permutations) with accuracy up to inverse digraphs. We advance outstanding open problem conjectured in JDEA paper on the structure of the second minimal odd orbits, which are defined as those that immediately follow the minimal orbits with respect to Sharkovski ordering. We pursue full analysis of the second minimal 7-orbits. It is proved that there are 11 types of second minimal 7-orbits with accuracy up to inverses. We apply this result to the problem on the distribution of superstable periodic windows within the chaotic regime of the bifurcation diagram of the one-parameter family of logistic type unimodal maps. It is revealed that by fixing the maximum number of appearances of the periodic windows there is a universal pattern of distribution. For example, by employing the notation n_i for the i th appearance of the n -orbit, all the superstable odd orbits up to 8th appearances while increasing the parameter are distributed according to the universal law

$$\begin{aligned} \dots \rightarrow (2k + 3)_1 \rightarrow (2k + 9)_5 \rightarrow (2k + 7)_3 \rightarrow (2k + 9)_6 \rightarrow (2k + 5)_2 \rightarrow \\ \rightarrow (2k + 9)_7 \rightarrow (2k + 7)_4 \rightarrow (2k + 9)_8 \rightarrow (2k + 1)_1 \rightarrow \dots \end{aligned} \quad (1)$$

where the branches successfully follow from right to left as $k = 1, 2, \dots$. The same universal route is continued to the left for all 8 appearances of the $2^n(2k + 1)$ -orbits successfully for positive integers $n = 1, 2, \dots$, and with the same order of appearance indices. Every orbit in (1) is universal, in the sense that it has a unique cyclic permutation and digraph independent of the unimodal map. In particular, the first appearance of all the orbits is always a minimal orbit, with precisely Type 1 digraph. It is observed that the second appearance of 7-orbit is a second minimal 7-orbit with Type 1 digraph. The reason for the relevance of the Type 1 minimal orbit is the fact that the topological structure of the unimodal map with single maximum is equivalent to the structure of the Type 1 piecewise linear endomorphism. Yet another important development of this research is the revelation of the pattern of the pattern dynamics with respect to increased number of appearances. Understanding the nature and characteristics of this fascinating universal route is an outstanding open problem for future investigations.