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ANALYSIS OF INTERFACES FOR THE NONLINEAR DIFFUSION EQUATION WITH LINEAR CONVECTION

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Abstract of Report Talk: We investigate the problem of interface development in a Cauchy problem for the nonlinear diffusion-convection equation

$$u_t = (u^m)_{xx} + bu_x, x \in \mathbb{R}, t > 0; \quad u(x, 0) = C(-x)_+^\alpha, x \in \mathbb{R},$$

where $m, \alpha, C > 0, b \in \mathbb{R}$. This problem arises in physics, biology, and chemistry; examples of applications include heat radiation in plasma, the spatial spread of biological populations, and the diffusion of chemicals through groundwater. The physics of the situation indicates that the direction of the movement of the interface is an outcome of the competition between nonlinear diffusion vs. linear convection. The problem of determining the short-time behavior for interfaces of nonlinear diffusion equations, known as a Barenblatt problem, was first formulated in the 1950s. A full solution of the Barenblatt problem for the reaction-diffusion equation was given in 2000 [Abdulla and King, *SIAM J. Math. Anal.*, 32, 2(2000), 235-260] and 2002 [Abdulla, *Nonlinear Analysis*, 50, 4(2002), 541-560], but the problem remains open for the reaction-diffusion-convection equation. It is proved that for the opposing direction of convection ($b > 0$) depending on m, α and C , the interface may initially expand or shrink. For the slow diffusion case ($m > 1$), the borderline case in the parameter space (m, α) is given through the curve $\alpha = 1/(m - 1)$. The interface expands if $\alpha < 1/(m - 1)$ and shrinks if $\alpha > 1/(m - 1)$. The behavior of the interface in the borderline case depends on the constant C . There is a critical value C_* such that the interface expands if $C > C_*$ and shrinks if $C < C_*$. In the latter case, the explicit global traveling wave solution is found. We identify the region in the parameter space where a global self-similar solution exists, and in particular, the direction of the interface changes in time: a so called turning interface phenomenon is observed. For the cooperating direction of the convection ($b < 0$), the interface always expands and an explicit formula for the interface and local solution is derived in the whole parameter space. For the fast diffusion case $m < 1$, there is an infinite speed of propagation. In this case, we derive that the asymptotics of the solution at infinity agree with those of the diffusion equation. A WENO numerical scheme was applied to the problem and numerical results support our proved estimations.

[Joint work with Jonathan Goldfarb and Kev Johnson]

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