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On the Structure of Minimal $4(2k+1)$-orbits of Continuous Endomorphisms and Universality in Chaos

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Abstract of Report Talk: We consider the problem on the structure of the periodic orbits of period $4(2k+1)$, $k=1,2,...$ of the continuous endomorphisms on the real line which are minimal with respect to Sharkovski ordering. By developing the new method suggested recently in *Abdulla et al. J. of Diff. Equat. and Appl., 19,8(2013), 1395-1416,* it is proved that independent of k, there are 64 types of digraphs (and cyclic permutations) with accuracy up to inverse digraphs. The proof is constructive and we accomplish the construction of all 64 types of digraphs and cyclic permutations for minimal 12-orbits. We apply this result to the problem on the distribution of periodic windows within the chaotic regime of the bifurcation diagram of the one-parameter family of logistic type unimodal maps. First, we confirm through numerical analysis the conjecture made in a recent JDEA paper that the first two appearances of all the $2^n(2k+1)$-periodic windows with $k \geq 3$, as well as first appearances of $5 \cdot 2^n$- and $3 \cdot 2^n$-orbits while increasing the parameter are distributed according to the universal law

$$\cdots \to 2^n \cdot 11 \to 2^n \cdot 7 \to 2^n \cdot 9 \to 2^n \cdot 5 \to 2^n \cdot 7 \to 2^n \cdot 3 \to$$
$$\cdots \to 11 \to 7 \to 9 \to 5 \to 7 \to 3 \to \dots \quad (1)$$

where the branches successfully follow from right to left as $n = 1, 2, ...$ Every orbit in (1) is universal, in the sense that it has a unique cyclic permutation and digraph independent of the unimodal map. In particular, the first appearance of $4(2k+1)$-orbit is always a minimal orbit, with precisely Type 1 digraph. The reason for the relevance of the Type 1 minimal orbit is the fact that the topological structure of the unimodal map with single maximum is equivalent to the structure of the Type 1 piecewise linear endomorphism. Yet another revelation of this research is the refinement of the universal law (1) for the third and fourth appearances of the periodic orbits. By employing the notation n_i for the ith appearance of the n-orbit, the following universal distribution of all the odd periodic orbits is revealed:

$$\dots 17_2 \to 19_4 \to 13_1 \to 17_3 \to 15_2 \to 17_4 \to 11_1 \to 15_3 \to 13_2 \to 15_4 \to 9_1 \to$$
$$13_3 \to 11_2 \to 13_4 \to 7_1 \to 11_3 \to 9_2 \to 11_4 \to 5_1 \to 9_3 \to 7_2 \to 9_4 \to 3_1 \to \dots$$

The same universal route is continued to the left for all four appearances of the $2^n(2k+1)$-orbits successfully for positive integers $n = 1, 2, ...$ Understanding the nature and characteristics of this fascinating universal route is an outstanding open problem for future investigations.

[Joint work with Anders Ruden]

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